Mathematical Proofs

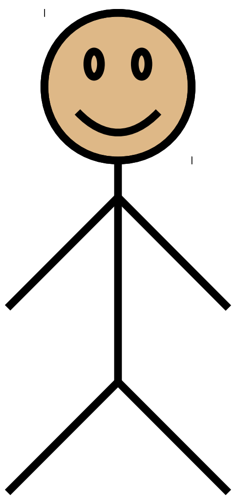
Outline for Today

- ***How to Write a Proof***
  - Synthesizing definitions, intuitions, and conventions.
- ***Proofs on Numbers***
  - Working with odd and even numbers.
- ***Universal and Existential Statements***
  - Two important classes of statements.
- ***Variable Ownership***
  - Who owns what?

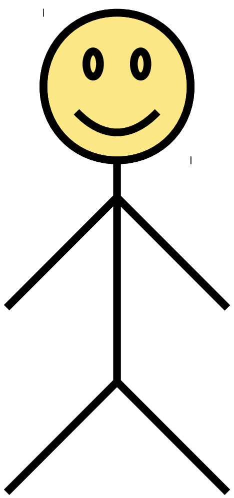
What is a Proof?

Proof as Dialog

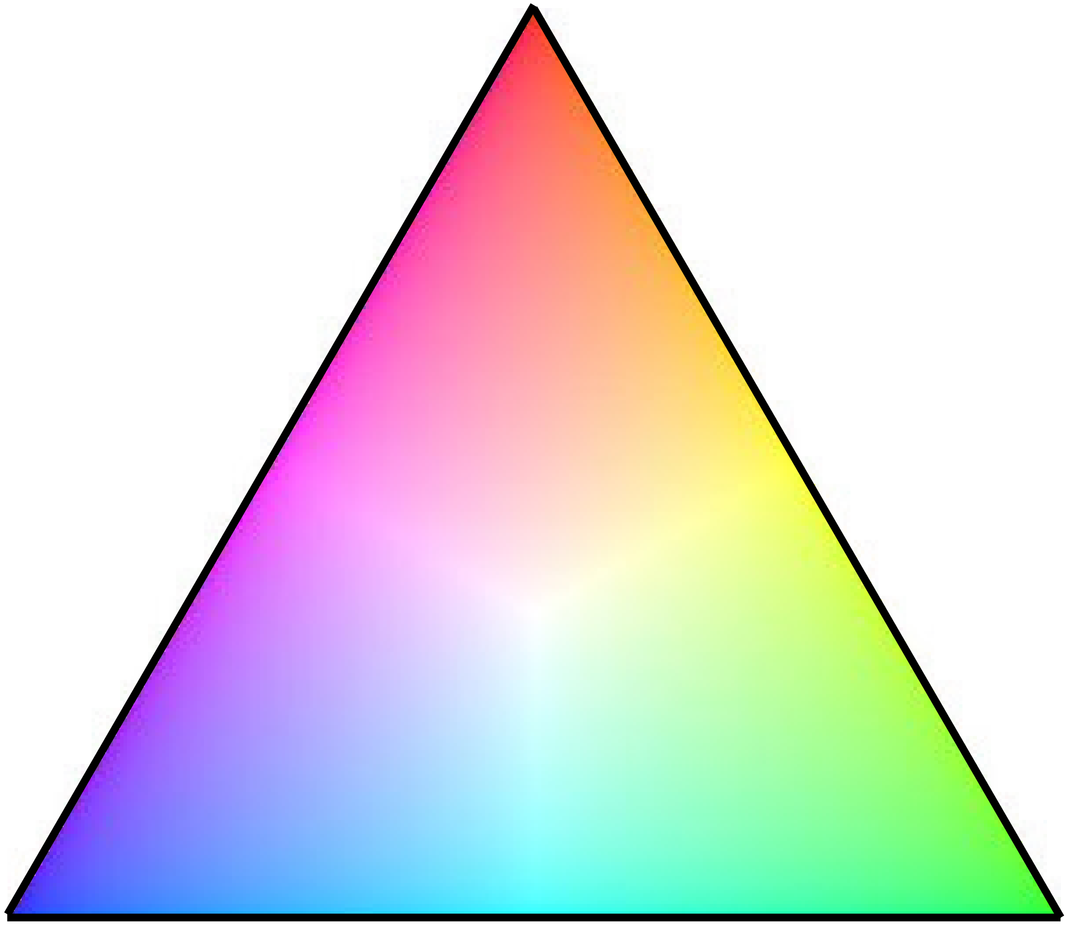
- A mathematical proof is a dialog between two parties: a ***proof writer*** and a ***proof reader***.
  - The ***proof writer*** knows a mathematical fact.
  - The ***proof reader*** is honest but skeptical.
- The proof writer’s job is to take the reader on a journey from ignorance to understanding.



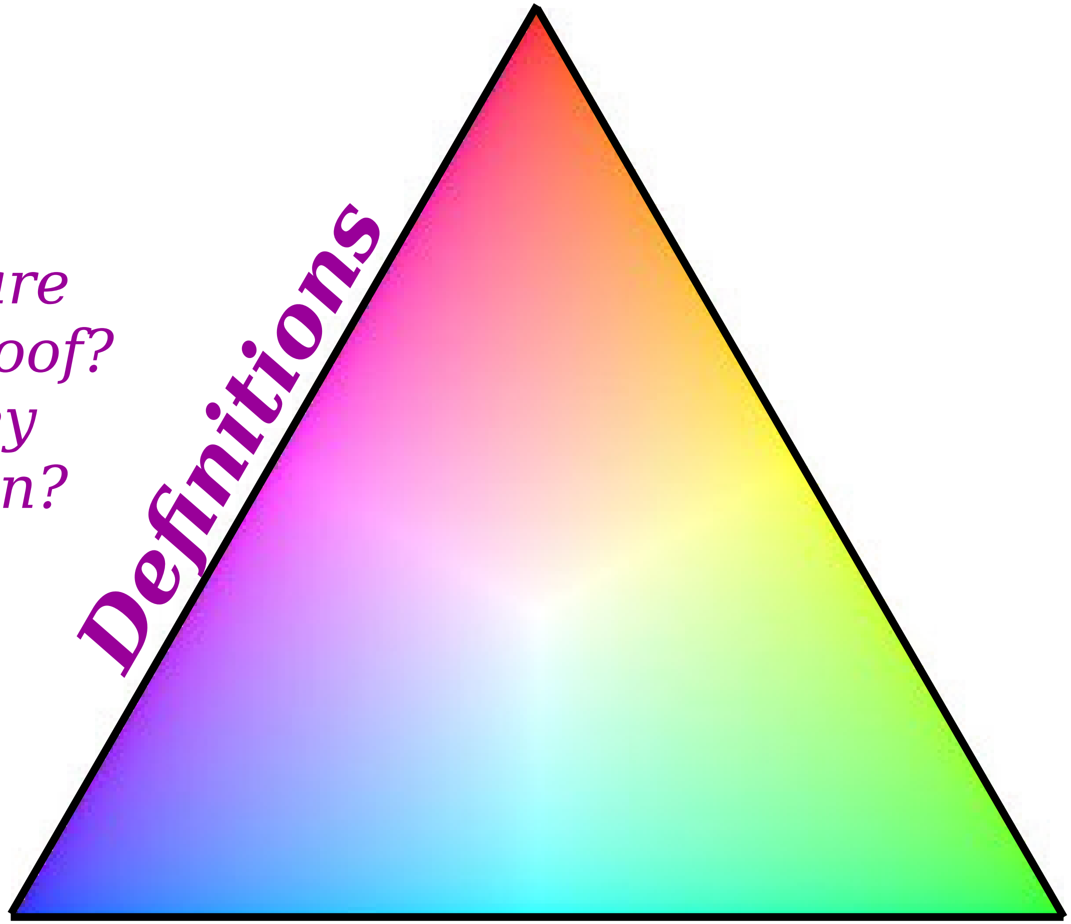
Proof Writer (You)

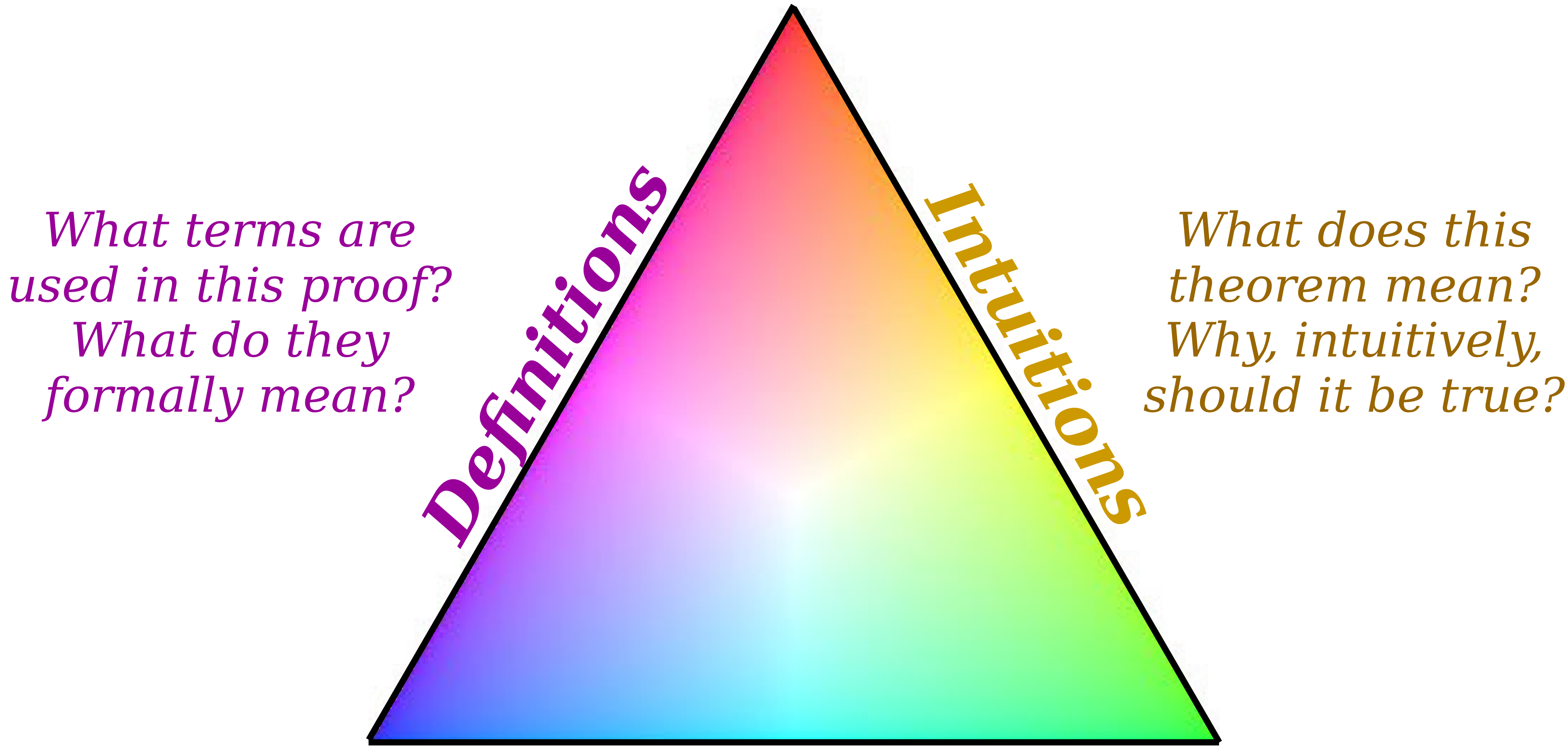


Proof Reader



What terms are used in this proof? What do they formally mean?

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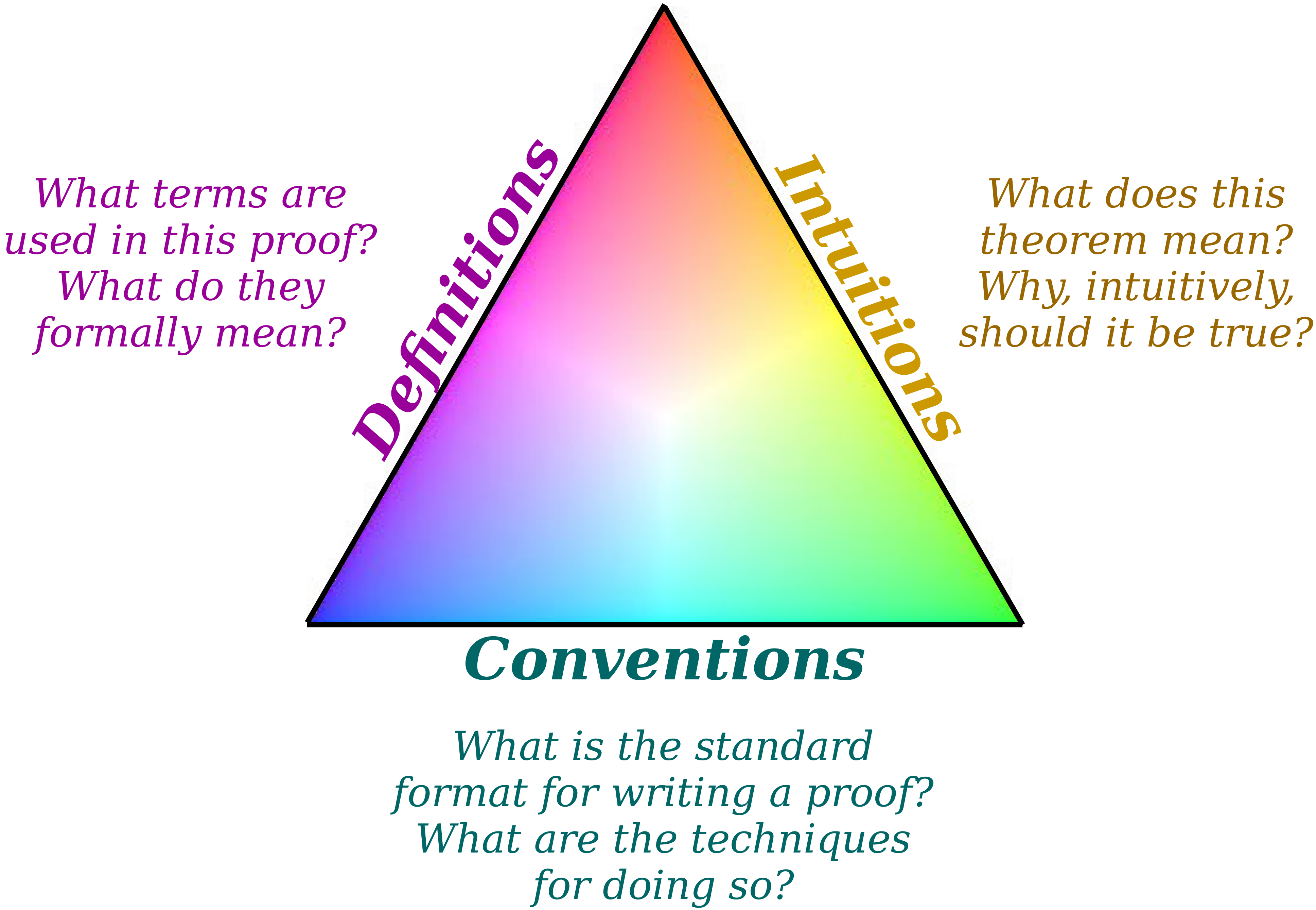
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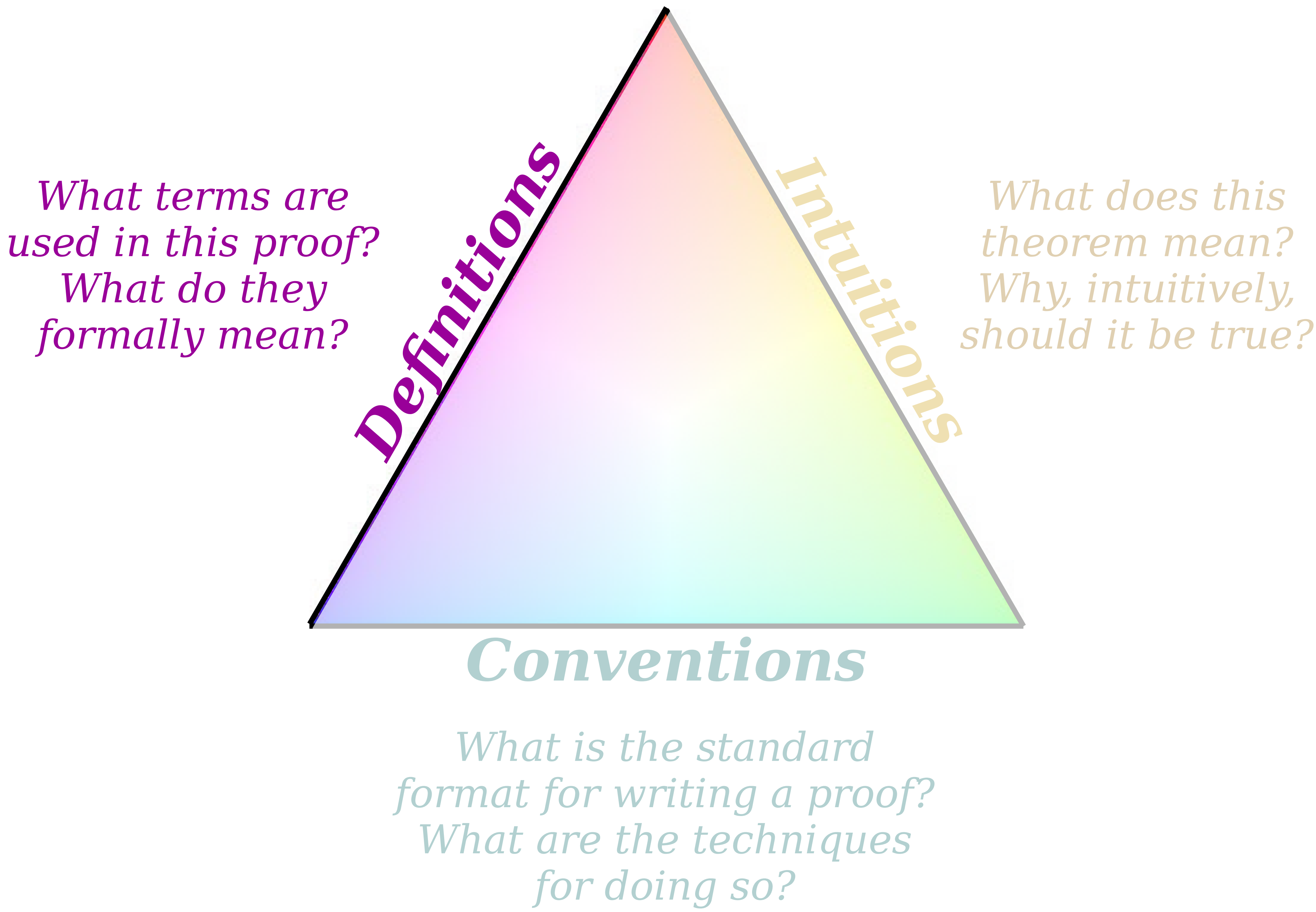
Writing our First Proof

Theorem: If n is an even integer, then n^2 is even.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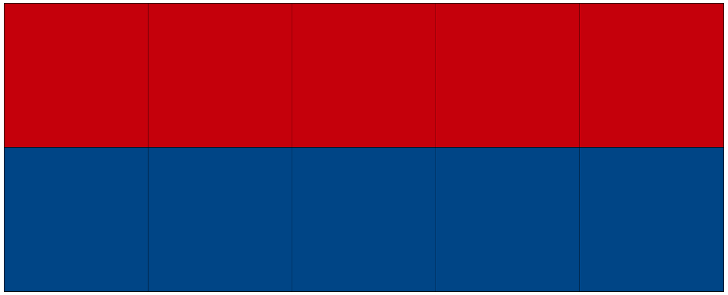
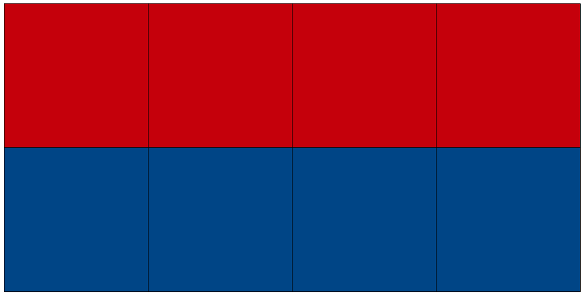
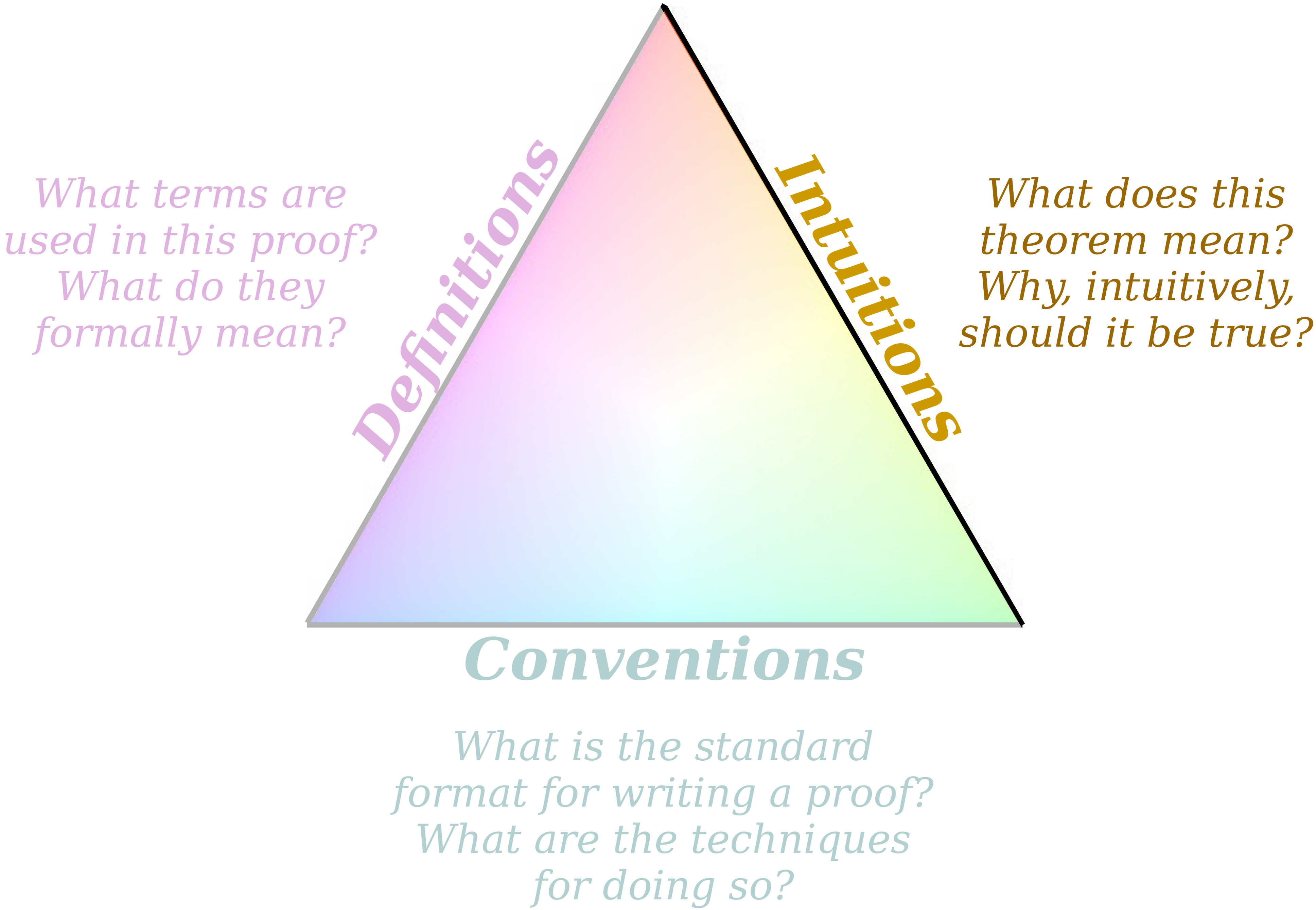
10  $2 \cdot \mathbf{5}$

8  $2 \cdot \mathbf{4}$

0 $2 \cdot \mathbf{0}$

An integer n is called ***even*** if there is an integer k where $n = 2k$.

Theorem: If n is an even integer, then n^2 is even.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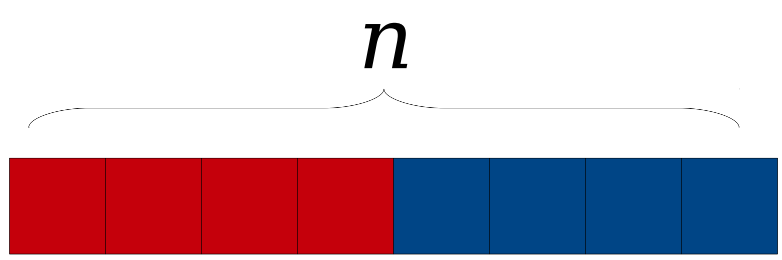
Let’s Try Some Examples!

$$
\begin{aligned}
2^2 &= 4 &&= 2 \cdot \mathbf{2} \\
10^2 &= 100 &&= 2 \cdot \mathbf{50} \\
0^2 &= 0 &&= 2 \cdot \mathbf{0} \\
(-8)^2 &= 64 &&= 2 \cdot \mathbf{32} \\
n^2 & &&= 2 \cdot \mathbf{?}
\end{aligned}
$$

What’s the pattern? How do we predict this?

Theorem: If n is an even integer, then n^2 is even.

Let's Draw Some Pictures!



$n = 2k$

Theorem: If n is an even integer, then n^2 is even.

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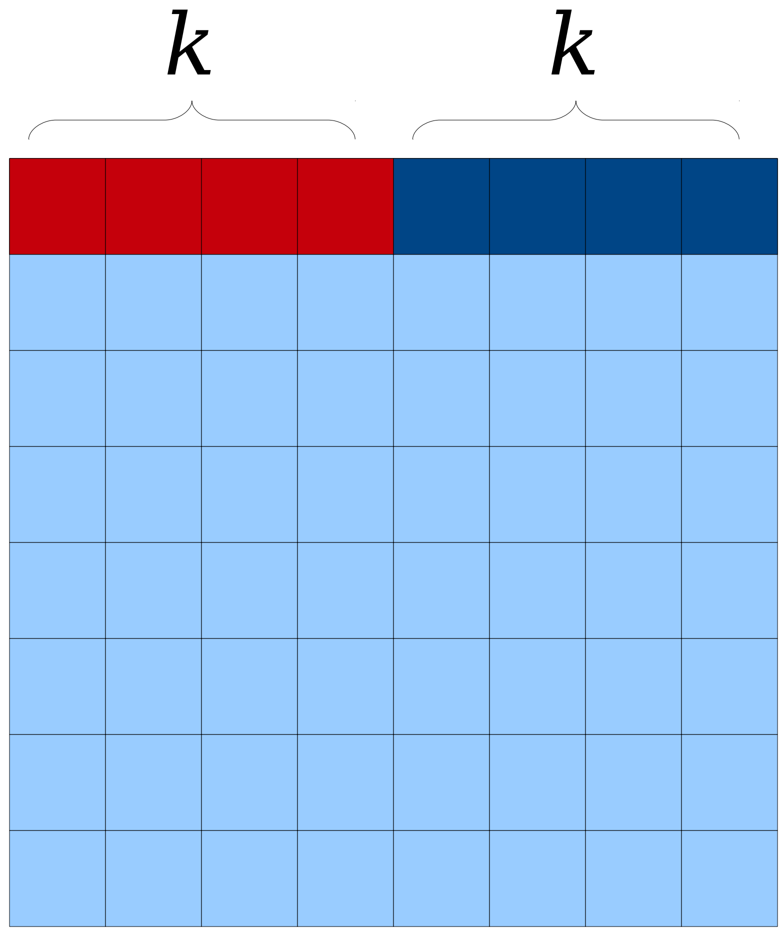

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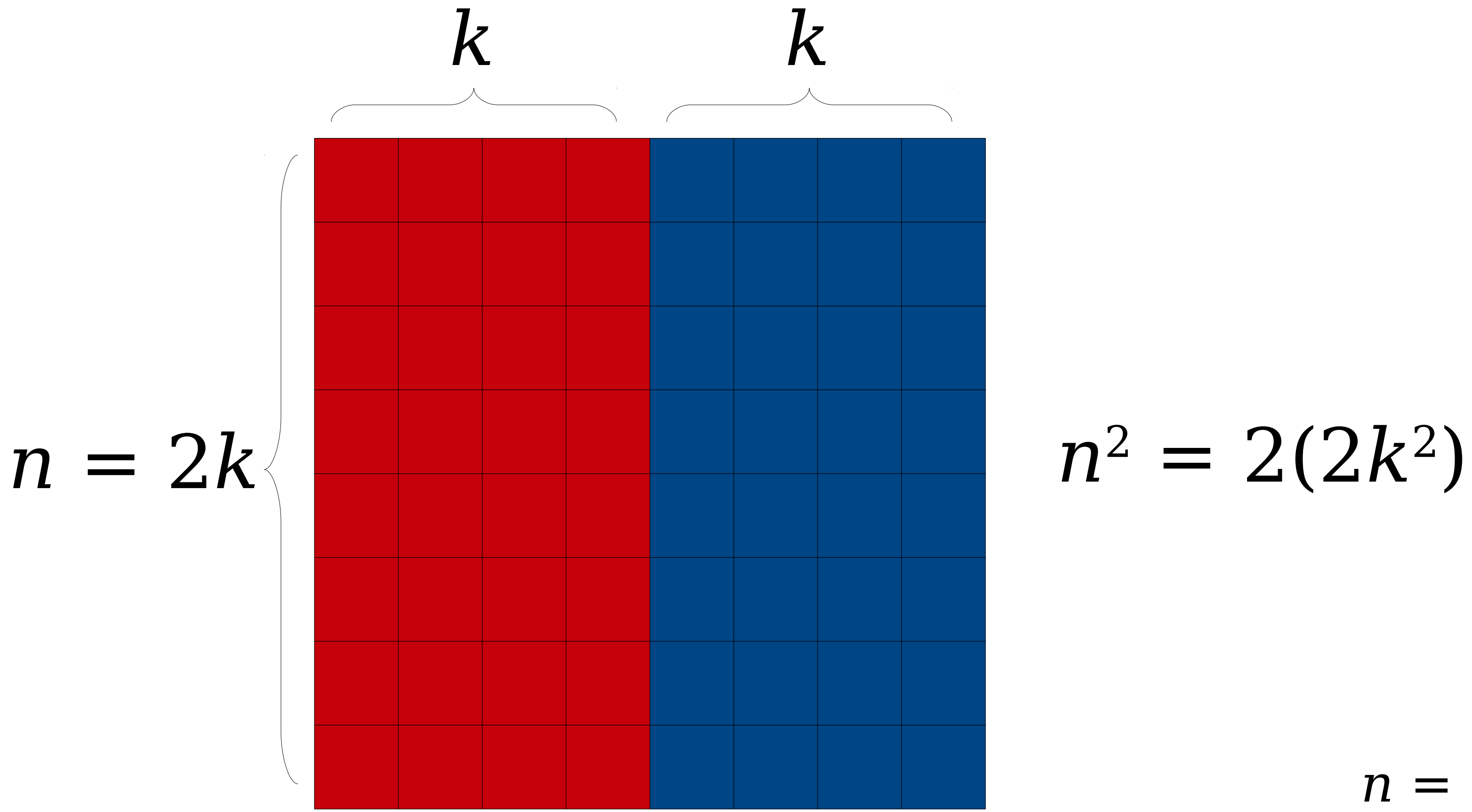

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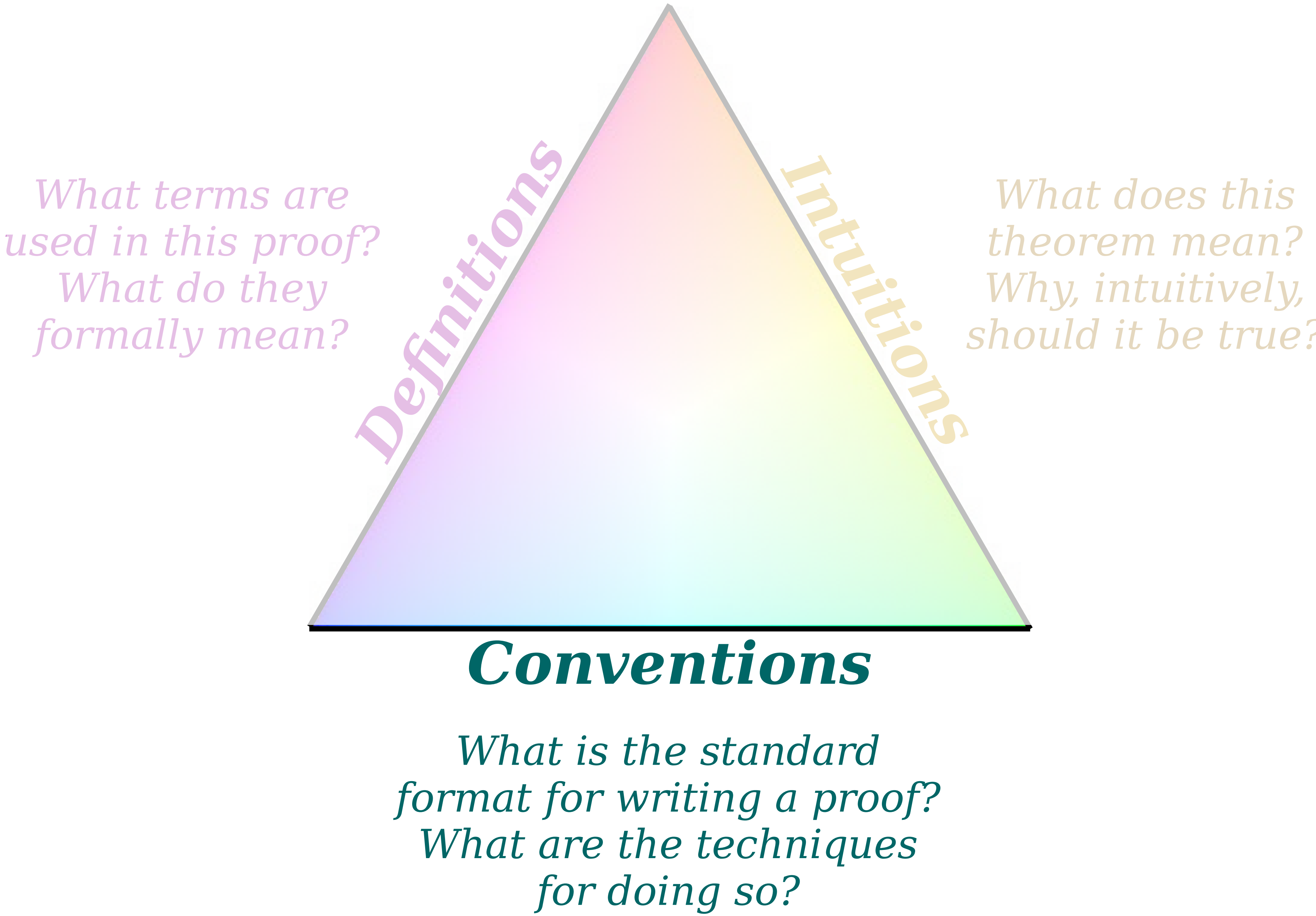
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Proof: Assume n is an even integer. We want to show that n^2 is even.

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$$n^2 = (2k)^2$$

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$$\begin{aligned} n^2 &= (2k)^2 \\ &= 4k^2 \end{aligned}$$

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$$\begin{aligned} n^2 &= (2k)^2 \\ &= 4k^2 \\ &= 2(2k^2). \end{aligned}$$

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> This symbol means "end of proof"

From this, we see that there is an integer m (namely, $2k^2$) where $n^2 = 2m$. Therefore, n^2 is even, which is what we wanted to show. $\blacksquare$

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To prove a statement of the form

"If *P* is true, then *Q* is true,"

start by asking the reader to assume that ***P*** is true.

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To prove a statement of the form

"If *P* is true, then *Q* is true,"

we assume ***P*** is true, then need to show that ***Q*** is true. Here, we're telling the reader where we're headed.

Our First Proof!

Theorem: If n is an even integer, then n^2 is even.

Proof: Assume n is an even integer. We want to show that n^2 is even.

Since n is even, there is some integer k such that $n = 2k$. This means that

$$n^2 = (2k)^2$$

This is the definition of an even integer. We need to use this definition to make this proof rigorous.

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Our First Proof!

Theorem: If

Proof: Assum
show that n

Since n is e
that $n = 2k$

Notice how we use the value of ***k*** that we obtained above. Giving names to quantities, allows us to manipulate them. This is similar to variables in programs.

$$
\begin{aligned}
n^2 &= (2k)^2 \\
&= 4k^2 \\
&= 2(2k^2).
\end{aligned}
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From this, we see that there is an integer m (namely, $2k^2$) where $n^2 = 2m$. Therefore, n^2 is even, which is what we wanted to show. ■

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Theorem: If n is an even integer, then n^2 is even.

Proof: Assume n is an even integer. We want to show that n^2 is even.

Since … h that n …

Our ultimate goal is to prove that n^2 is even. This means that we need to find some m where $n^2 = 2m$. Here, we're explicitly showing how we can do that.

From this, we see that there is an integer m (namely, $2k^2$) where $n^2 = 2m$. Therefore, n^2 is even, which is what we wanted to show. ■

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Since n is even, there is some integer k such that $n = 2k$. This means that

$$n^2 = (2k)^2 = 4k^2 = 2(2k^2)$$

Hey, that's what we said we were going to do! We're done.

From this, we see that there is an integer m (namely, $2k^2$) where $n^2 = 2m$. Therefore, n^2 is even, which is what we wanted to show. ■

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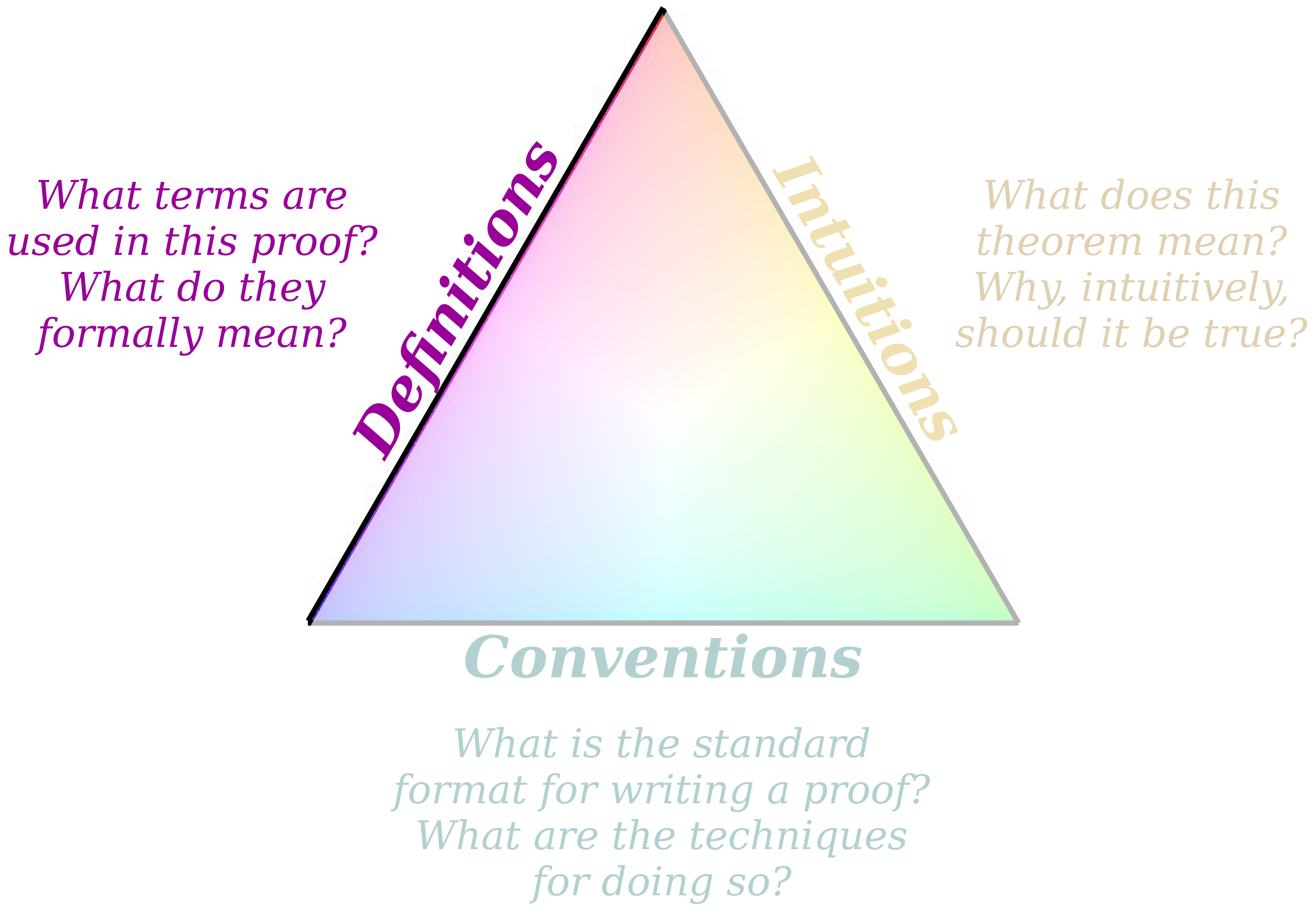
Our Next Proof

Theorem: For any integers m and n, if m and n are odd, then $m + n$ is even.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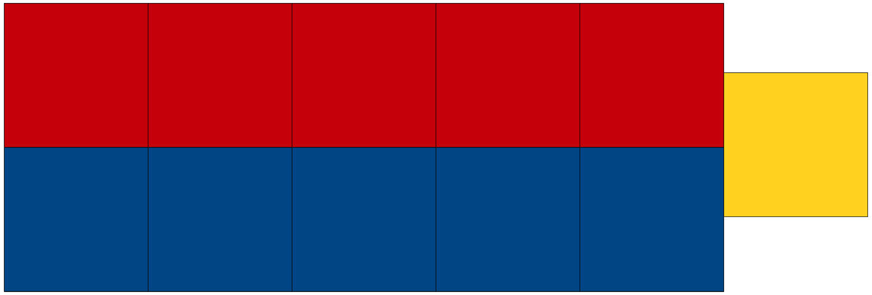
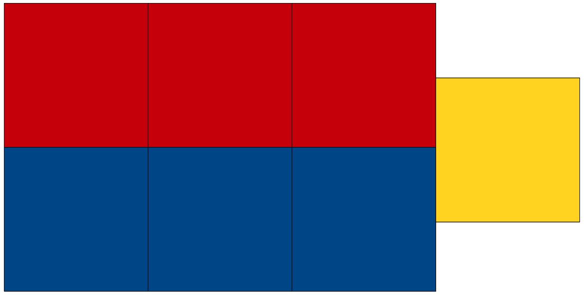
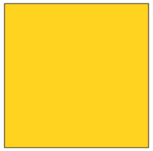
11  $2 \cdot \mathbf{5} + \mathbf{1}$

7  $2 \cdot \mathbf{3} + \mathbf{1}$

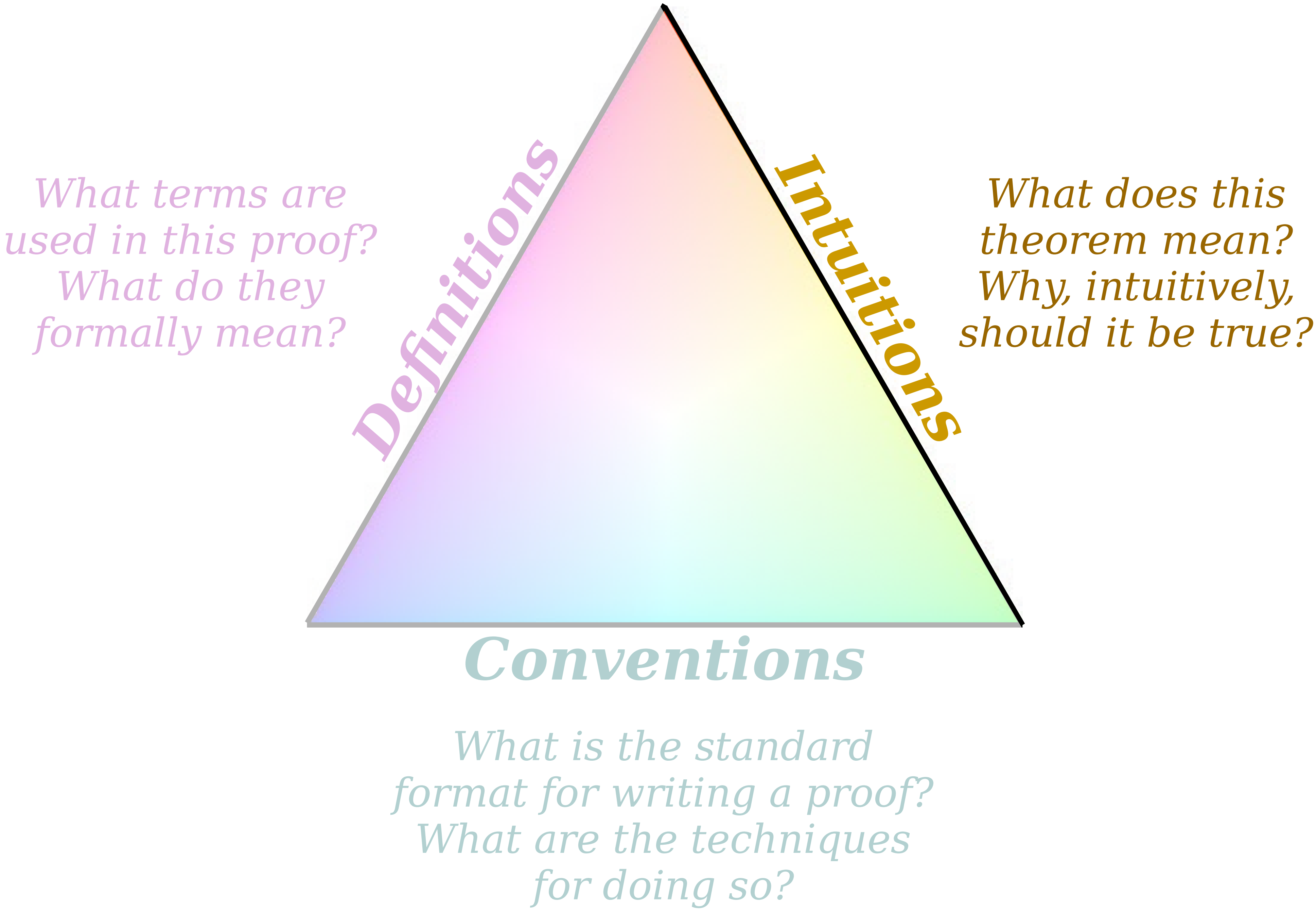
1  $2 \cdot \mathbf{0} + \mathbf{1}$

An integer n is called ***odd*** if
there is an integer k where $n = 2k+1$.

Going forward, we'll assume the following:

1. Every integer is either even or odd.
2. No integer is both even and odd.

Theorem: For any integers m and n, if m and n are odd, then $m + n$ is even.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Let's Try Some Examples!

$$1 + 1 = 2 = 2 \cdot \mathbf{1}$$

$$137 + 103 = 240 = 2 \cdot \mathbf{120}$$

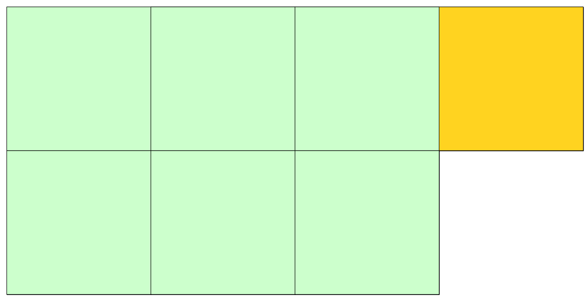
$$-5 + 5 = 0 = 2 \cdot \mathbf{0}$$

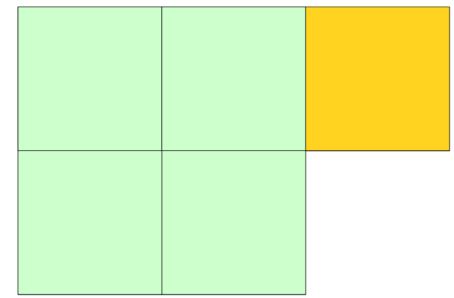
$$m + n = 2 \cdot \mathbf{?}$$

What's the pattern? How do we predict this?

Theorem: For any integers m and n, if m and n are odd, then $m+n$ is even.

Let’s Draw Some Pictures!





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Let's Do Some Math!

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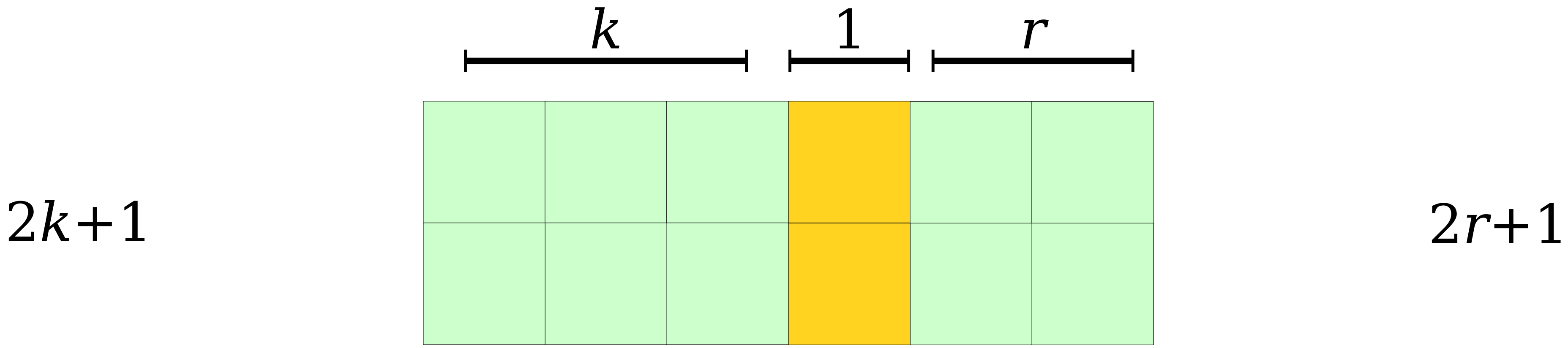
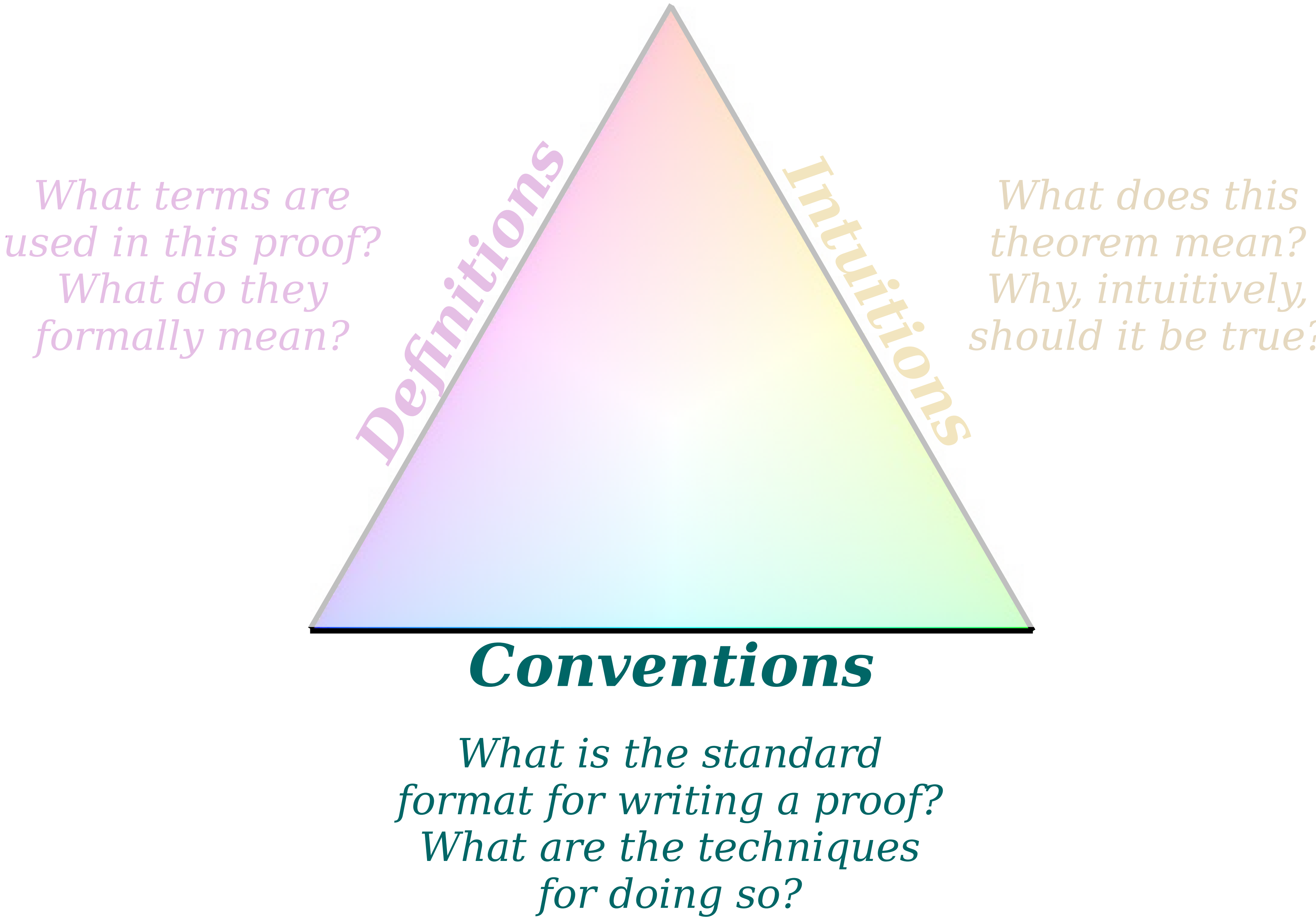
Let's Do Some Math!



$$(2k+1) + (2r+1) = 2(k + r + 1)$$

Theorem: For any integers m and n, if m and n are odd, then $m+n$ is even.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Proof: Consider any arbitrary integers m and n where m and n are odd. We need to show that $m + n$ is even.

Since m is odd, we know that there is an integer k where

$$m = 2k + 1. \qquad (1)$$

Theorem: For any integers m and n, if m and n are odd, then $m + n$ is even.

Proof: Consider any arbitrary integers m and n where m and n are odd. We need to show that $m + n$ is even.

Since m is odd, we know that there is an integer k where

$$m = 2k + 1. \tag{1}$$

Similarly, because n is odd there must be some integer r such that

$$n = 2r + 1. \tag{2}$$

Theorem: For any integers m and n, if m and n are odd, then $m + n$ is even.

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By adding equations (1) and (2) we learn that

$$m + n = 2k + 1 + 2r + 1$$

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By adding equations (1) and (2) we learn that

$$\begin{aligned} m + n &= 2k + 1 + 2r + 1 \\ &= 2k + 2r + 2 \end{aligned}$$

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$$\begin{aligned} m + n &= 2k + 1 + 2r + 1 \\ &= 2k + 2r + 2 \\ &= 2(k + r + 1). \end{aligned} \tag{3}$$

Equation (3) tells us that there is an integer s (namely, $k + r + 1$) such that $m + n = 2s$.

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We ask the reader to make an *arbitrary choice*. Rather than specifying what m and n are, we're signaling to the reader that they could, in principle, supply any choices of m and n that they'd like.

By letting the reader pick m and n arbitrarily, anything we prove about m and n will generalize to all possible choices for those values.

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By adding …

To prove a statement of the form

"If *P* is true, then *Q* is true,"

start by asking the reader to assume that ***P*** is true.

$$= 2k + 2r + 2$$

$$= 2(k + r + 1). \qquad (3)$$

Equation (3) tells us that there is an integer s (namely, $k + r + 1$) such that $m + n = 2s$. Therefore, we see that $m + n$ is even, as required. ■

Theorem: For any integers m and n, if m and n are odd, then $m + n$ is even.

Proof: Consider any arbitrary integers m and n where m and n are odd. We need to show that $m + n$ is even.

Since m i...

Similarly, ...ch that

By adding...

To prove a statement of the form

"If *P* is true, then *Q* is true,"

after assuming ***P*** is true, you need to show that ***Q*** is true.

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Numbering these equalities lets us refer back to them later on, making the flow of the proof a bit easier to understand.

Proof: Consider any

odd. We need to s

Since m is odd, w

$$m = 2k + 1. \qquad (1)$$

Similarly, because n is odd there must be some integer r such that

$$n = 2r + 1. \qquad (2)$$

By adding equations (1) and (2) we learn that

$$\begin{aligned} m + n &= 2k + 1 + 2r + 1 \\ &= 2k + 2r + 2 \\ &= 2(k + r + 1). \qquad (3) \end{aligned}$$

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(2)

… that

… $+ 1$

… . (3)

…ger s (namely, $k + r + 1$) … that $m + n$ is even, as

This is a complete sentence! Proofs are expected to be written in complete sentences, so you'll often use punctuation at the end of formulas.

We recommend using the "mugga mugga" test – if you read a proof and replace all the mathematical notation with "mugga mugga," what comes back should be a valid sentence.

Theorem: For any integers m and n, if m and n are odd, then $m + n$ is even.

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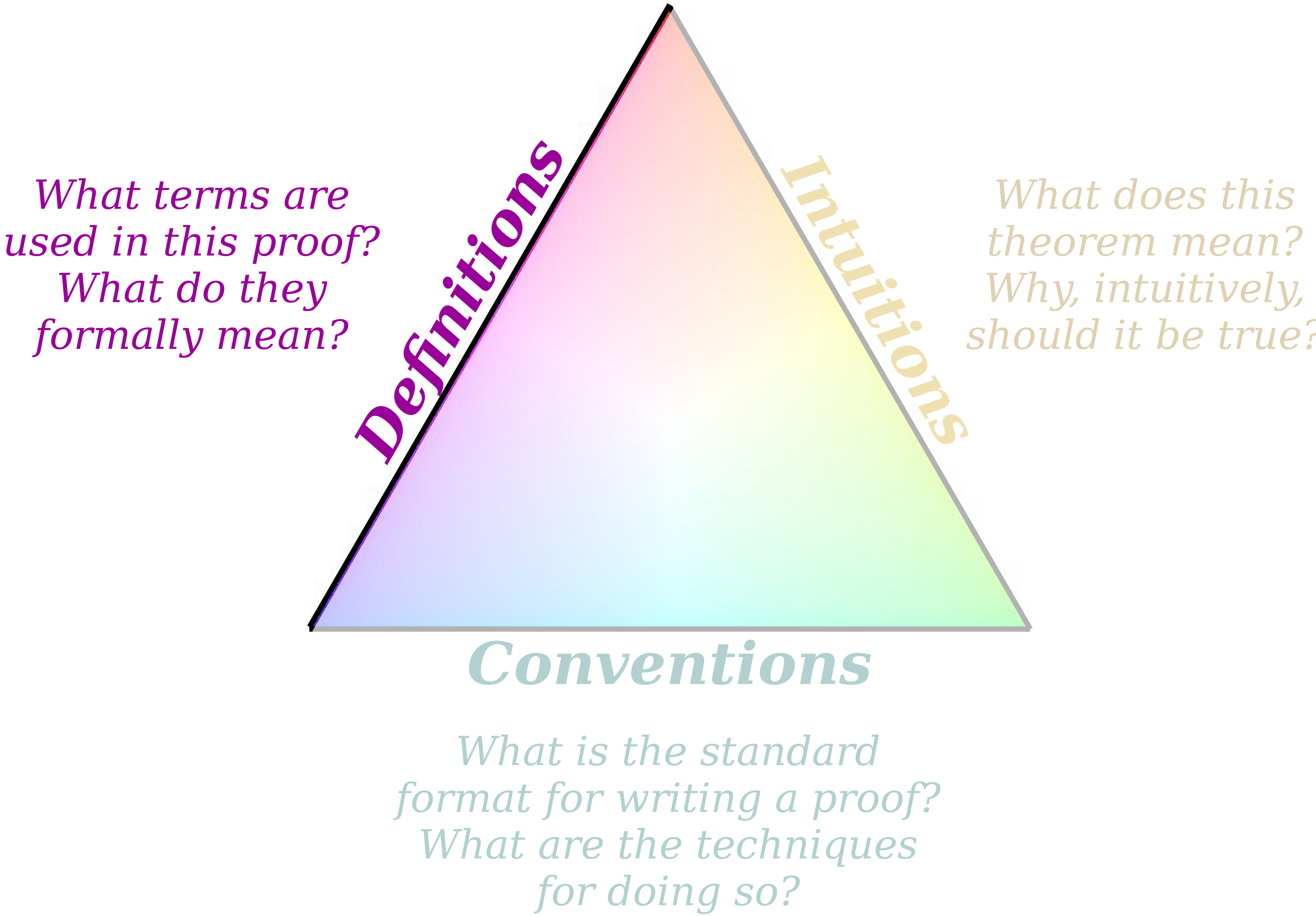
Some Little Exercises

- Here's a list of other theorems that are true about odd and even numbers:
  - ***Theorem:*** The sum and difference of any two even numbers is even.
  - ***Theorem:*** The sum and difference of an odd number and an even number is odd.
  - ***Theorem:*** The product of any integer and an even number is even.
  - ***Theorem:*** The product of any two odd numbers is odd.
- Going forward, we'll just take these results for granted. Feel free to use them in the problem sets.
- If you'd like to practice the techniques from today, try your hand at proving these results!

Universal and Existential Statements

Theorem: For any odd integer n, there exist integers r and s where $r^2 - s^2 = n$.


What terms are used in this proof? What do they formally mean?
Definitions
Intuitions
What does this theorem mean? Why, intuitively, should it be true?
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This result is true for every possible choice of odd integer n. It'll work for $n = 1$, $n = 137$, $n = 103$, etc.

Theorem: For any odd integer n, there exist integers r and s where $r^2 - s^2 = n$.

We aren't saying this is true for every choice of r and s. Rather, we're saying that *somewhere out there* are choices of r and s where this works.

Universal vs. Existential Statements

- A ***universally-quantified statement*** is a statement of the form

For all *x*, `[some-property]` holds for *x*.

- We've seen how to prove these statements.
- An ***existentially-quantified statement*** is a statement of the form

There is some *x* where `[some-property]` holds for *x*.

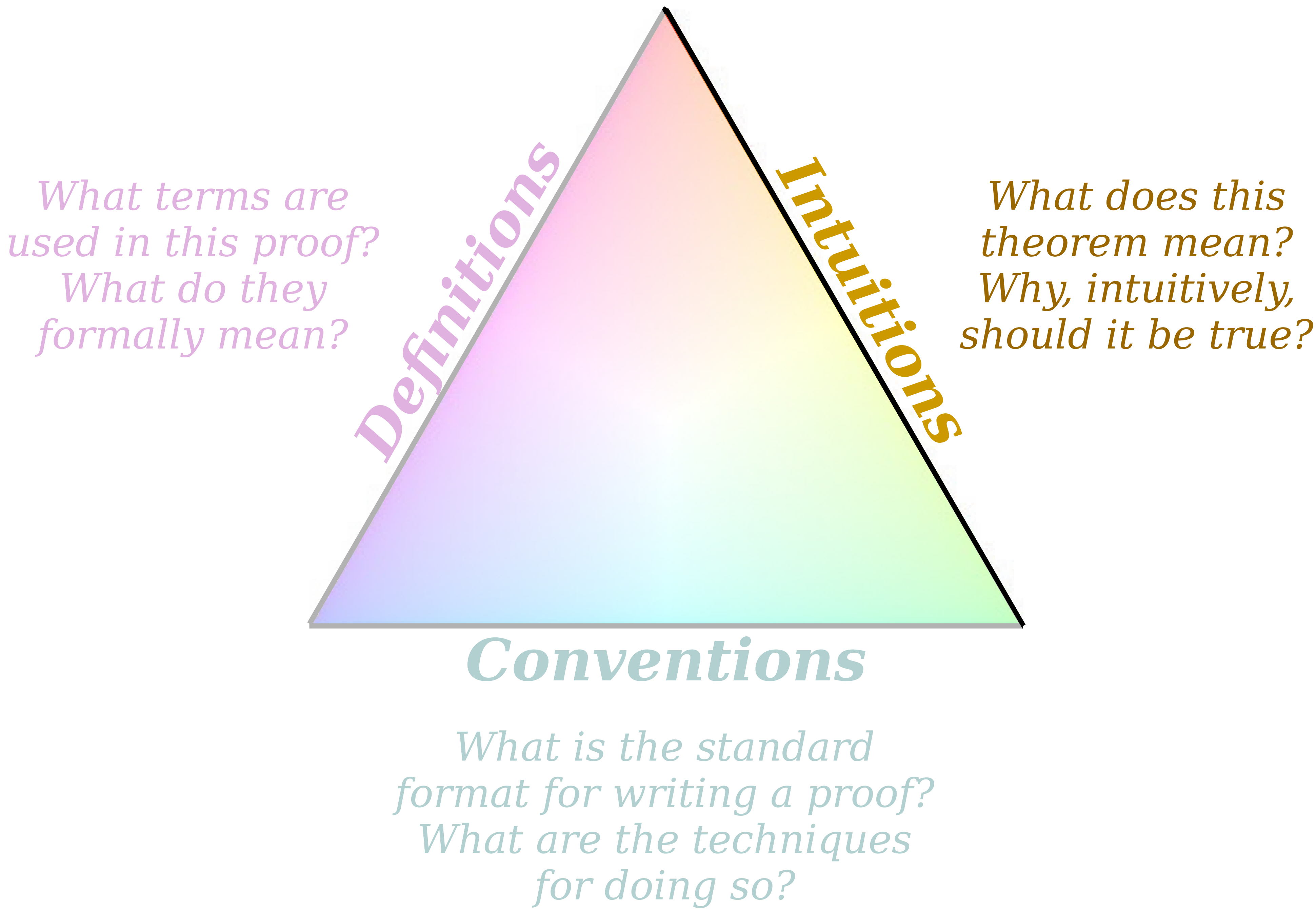
- How do you prove an existentially-quantified statement?

Proving an Existential Statement

- Over the course of the quarter, we will see several different ways to prove an existentially-quantified statement of the form

  There is an *x* where `[some-property]` holds for *x*.

- ***Simplest approach:*** Search far and wide, find an *x* that has the right property, then show why your choice is correct.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Let's Try Some Examples!

$1 = \_\_\_^2 - \_\_\_^2$

$3 = \_\_\_^2 - \_\_\_^2$

$5 = \_\_\_^2 - \_\_\_^2$

$7 = \_\_\_^2 - \_\_\_^2$

$9 = \_\_\_^2 - \_\_\_^2$

Theorem: For any odd integer n,
there exist integers r and s where $r^2 - s^2 = n$.

Let’s Try Some Examples!

$$1 = \mathbf{1}^2 - \mathbf{0}^2$$

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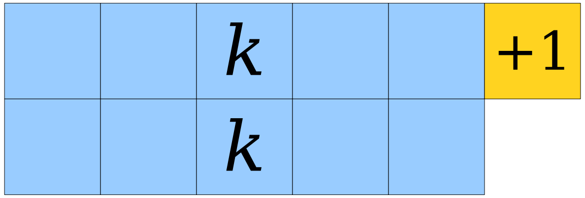
$\mathbf{4} + 1 = \mathbf{5}^2 - \mathbf{4}^2$

Educated Guess:

$2k + 1 = (k+1)^2 - k^2.$

Theorem: For any odd integer n, there exist integers r and s where $r^2 - s^2 = n$.

Let’s Draw Some Pictures!



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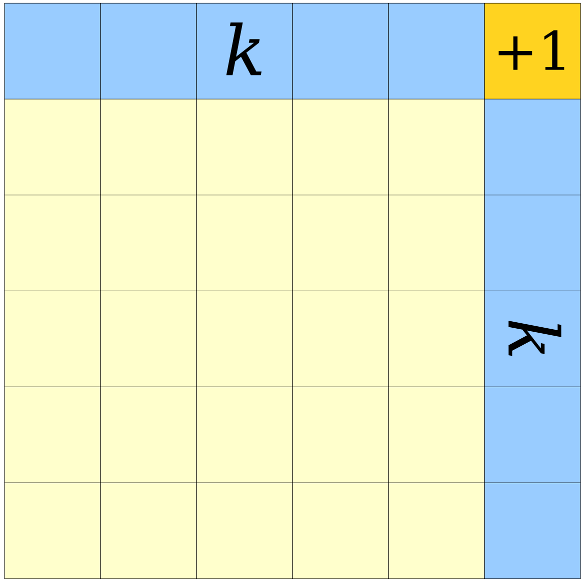
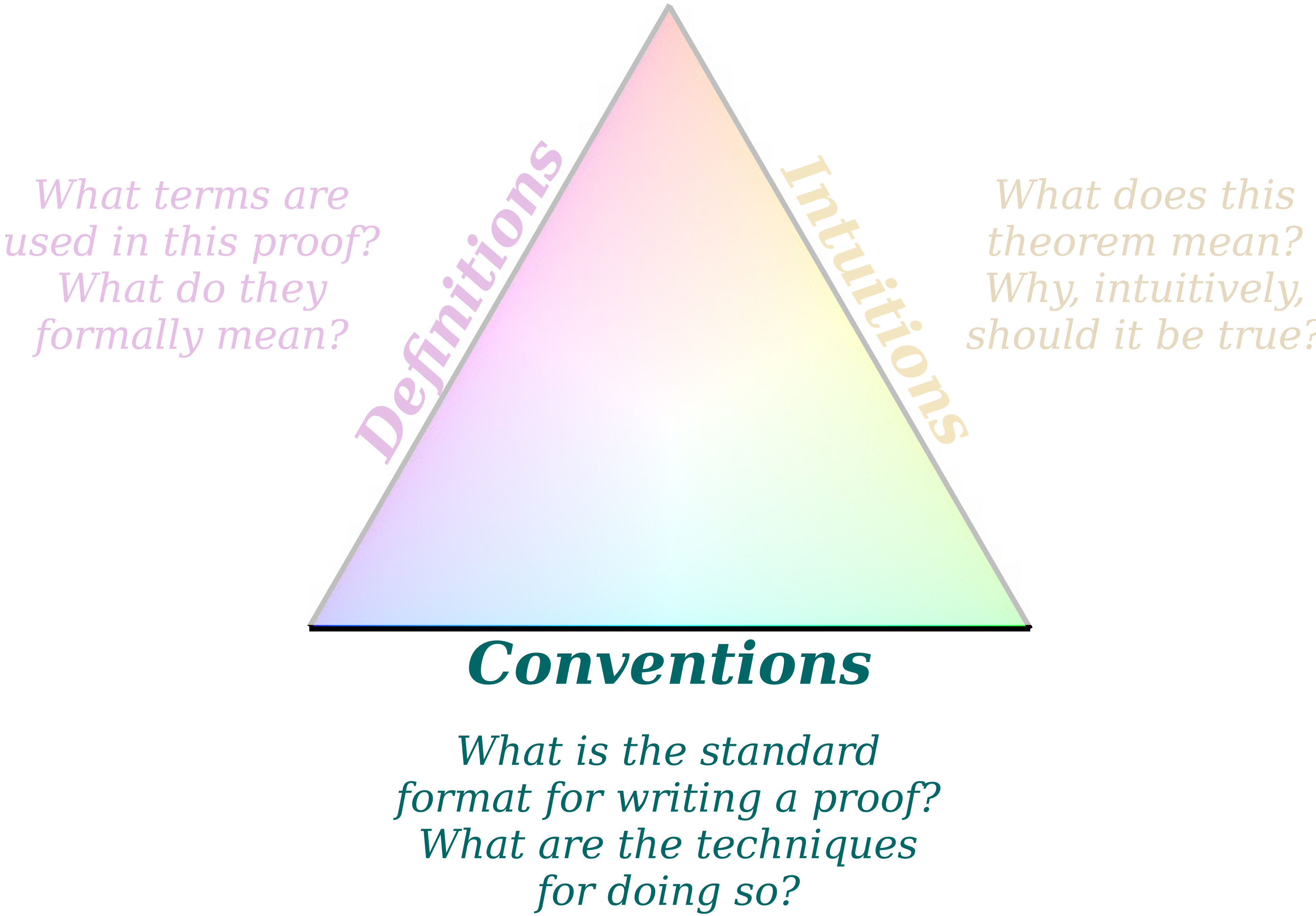


$$(k+1)^2 - k^2 = 2k+1$$

Theorem: For any odd integer n, there exist integers r and s where $r^2 - s^2 = n$.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Proof: Let n be an arbitrary odd integer. We will show that there exist integers r and s where $r^2 - s^2 = n$.

Since r [...] here $n = 2k$ [...] we see that

We ask the reader to make an *arbitrary choice*. Rather than specifying what ***n*** is, we're signaling to the reader that they could, in principle, supply any choice ***n*** that they'd like.

$$= \quad 2k + 1$$

$$= \quad n.$$

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$$r^2 - s^2 = (k+ \ldots$$
$$= k^2 + \ldots$$
$$= 2k + 1$$
$$= n.$$

As always, it's helpful to write out what we need to demonstrate with the rest of the proof.

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$r^2 - s^2 \quad = \quad (k+1)^2$

$= \quad k^2 + 2$

$= \quad 2k + 1$

$= \quad n.$

This means that $r^2 - s^2 = n$, w

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We're trying to prove an existential statement. The easiest way to do that is to just give concrete choices of the objects being sought out.

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Time-Out for Announcements!

Outline for Today

- Problem Set Zero is due this Friday at 4:00PM. It must be completed individually.
- After that, the remaining problem sets can be done individually or in pairs.
  - Each pair should make a single joint submission.
- We have advice about how to work efectively in pairs up on the course website – check the “Guide to Partners.”
- Want to work in a pair, but don’t know who to work with? You can:
  - Post a comment on the ***pinned partners thread*** on Ed.
  - Fill out ***this Google Form*** and we’ll connect you with a
  - partner on Monday.

Qt Creator Help Session

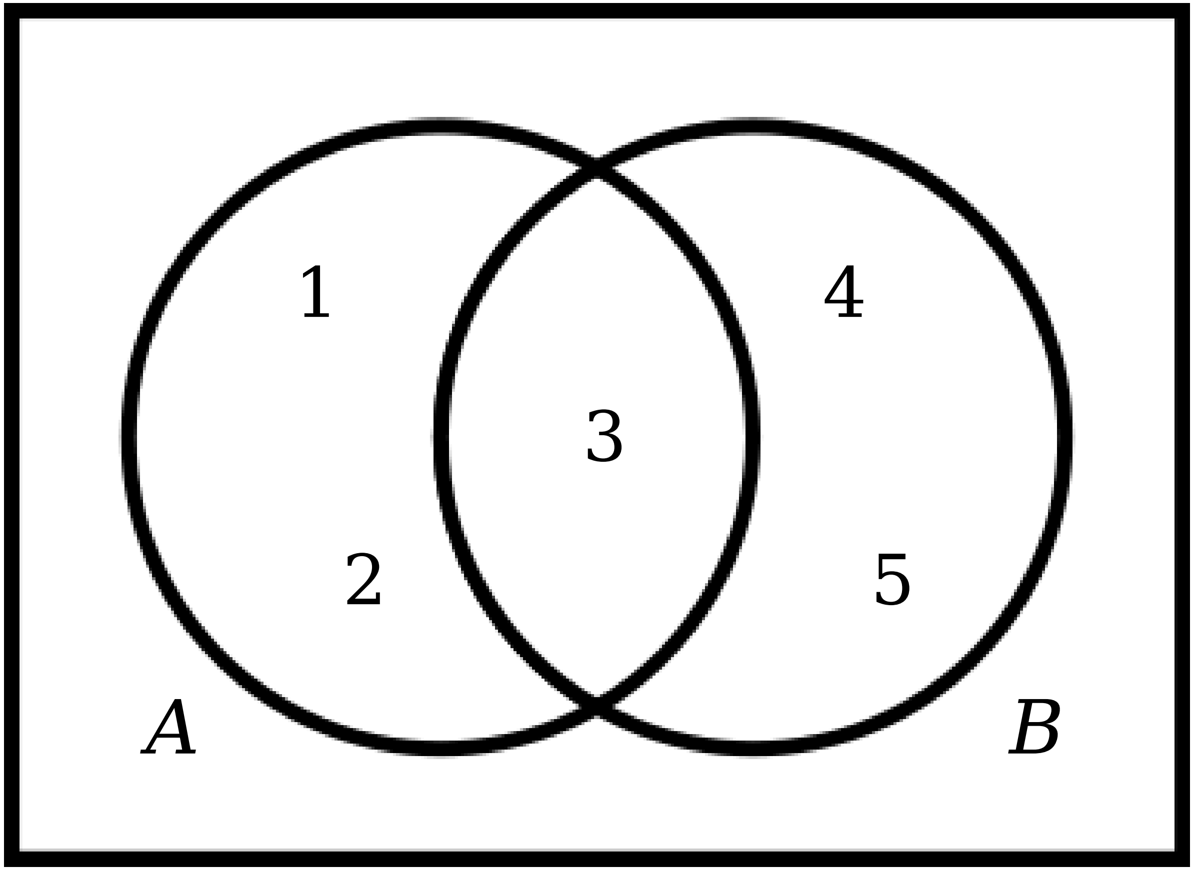
- The lovely CS106B staff are running a Qt Creator Help Session tomorrow evening if you're having trouble getting Qt Creator up and running on your system.
- Runs Thursday, January 11th from ***5:00PM – 7:00PM*** in Durand 353.
- SCPD students – please reach out to us if you need help setting things up. We'll do our best to help out.

Back to CS103!

A ***set*** is an unordered collection of distinct objects, which may be anything, including other sets.

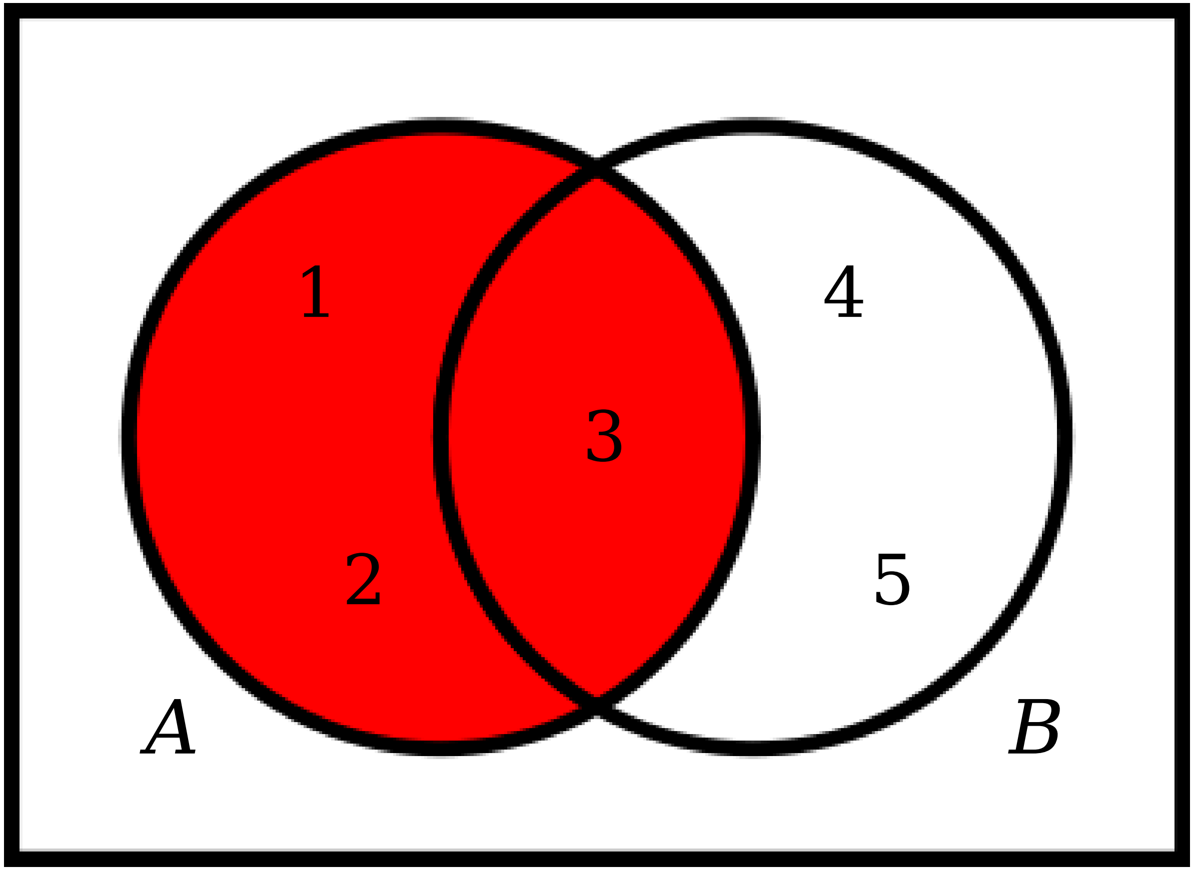
Combining Sets

Venn Diagrams



$A = \{ 1, 2, 3 \}$
$B = \{ 3, 4, 5 \}$

Venn Diagrams



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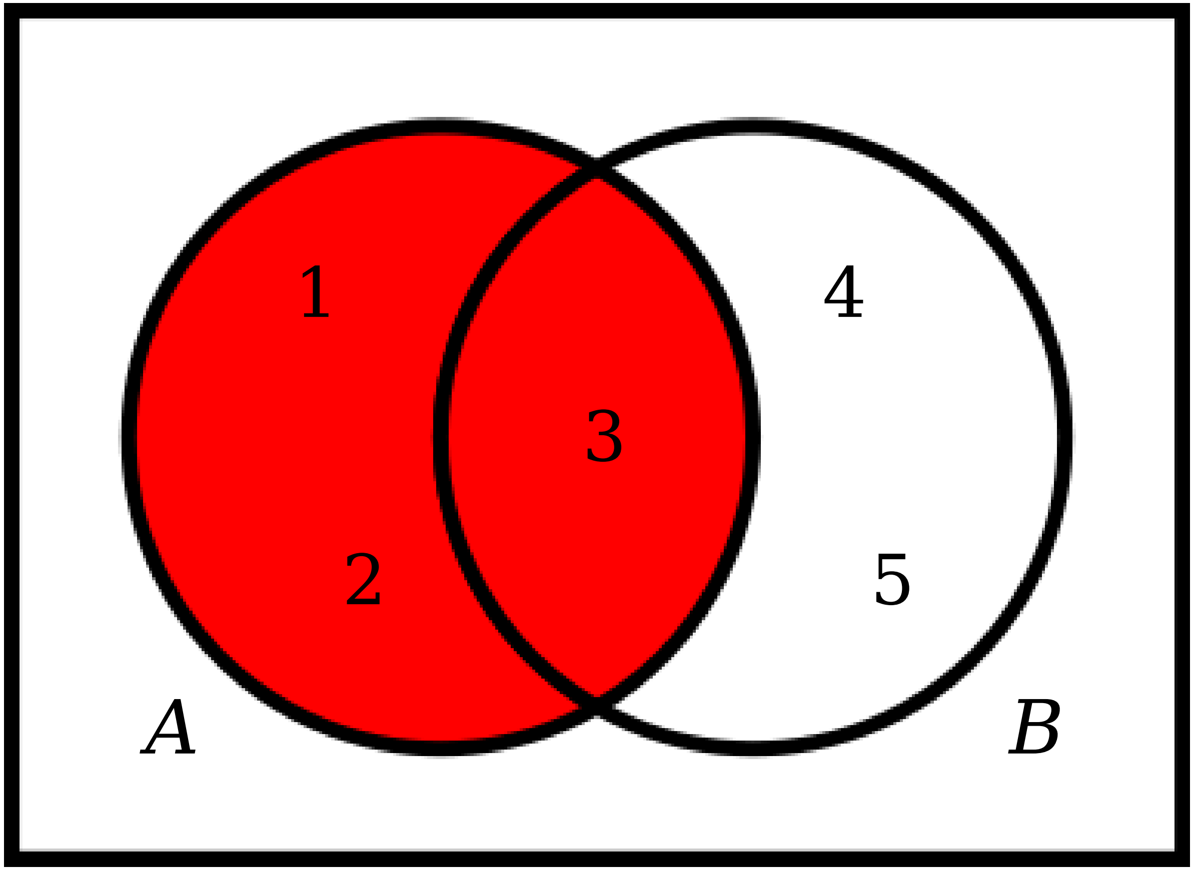

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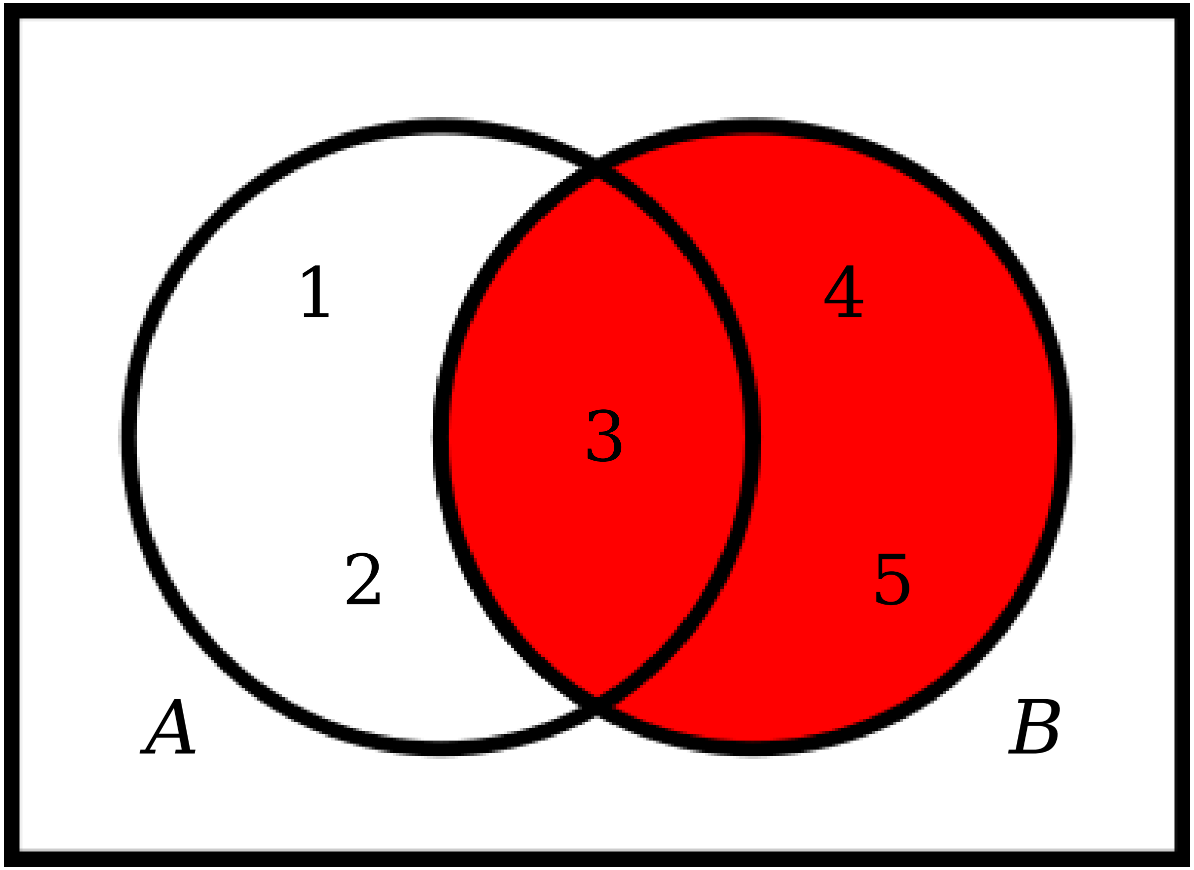

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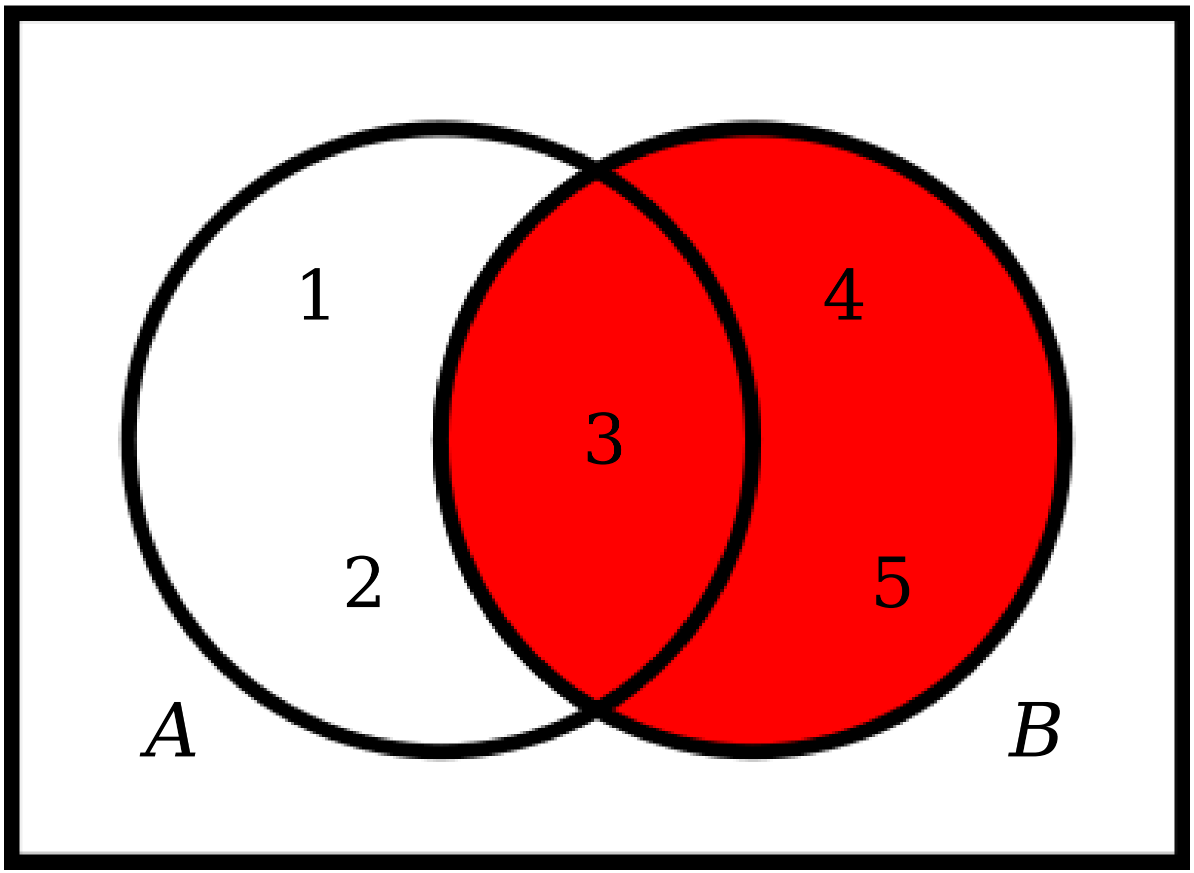

B

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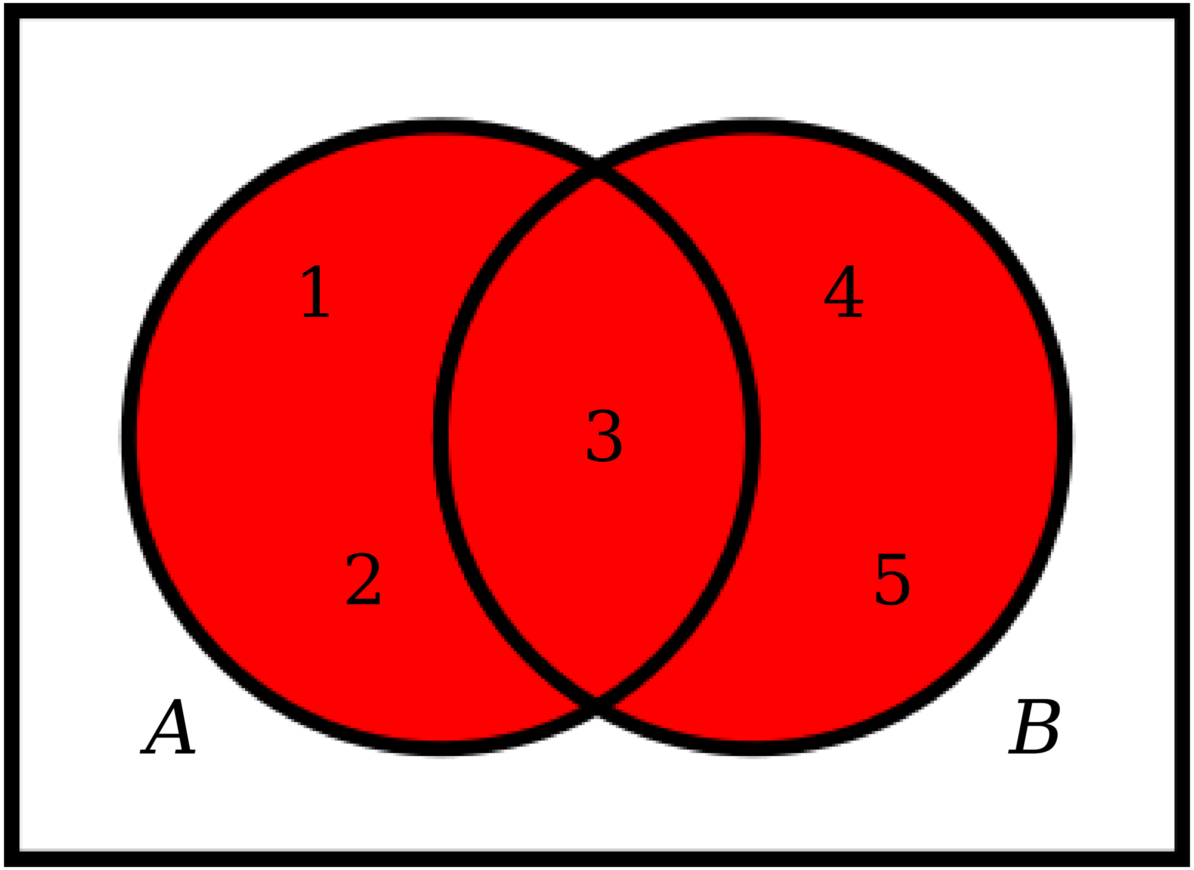
Venn Diagrams



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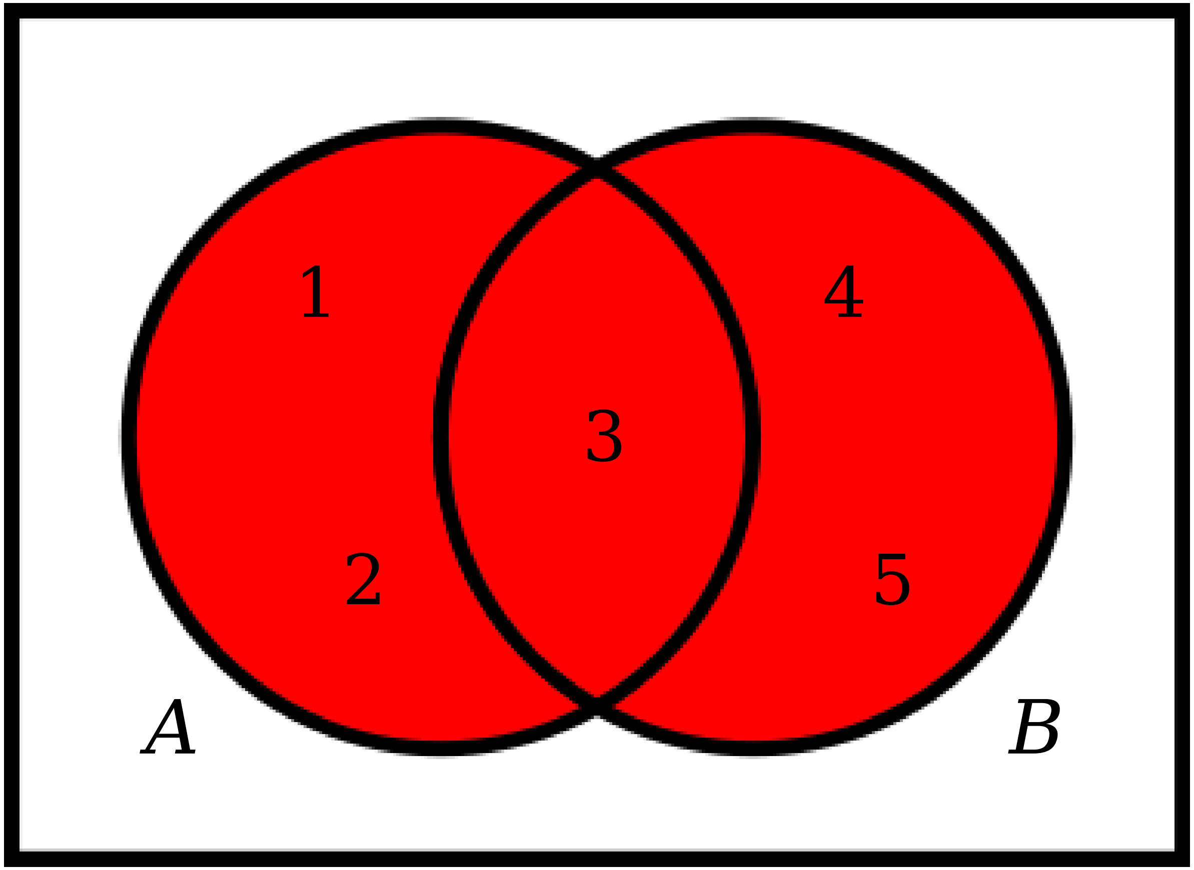


Union

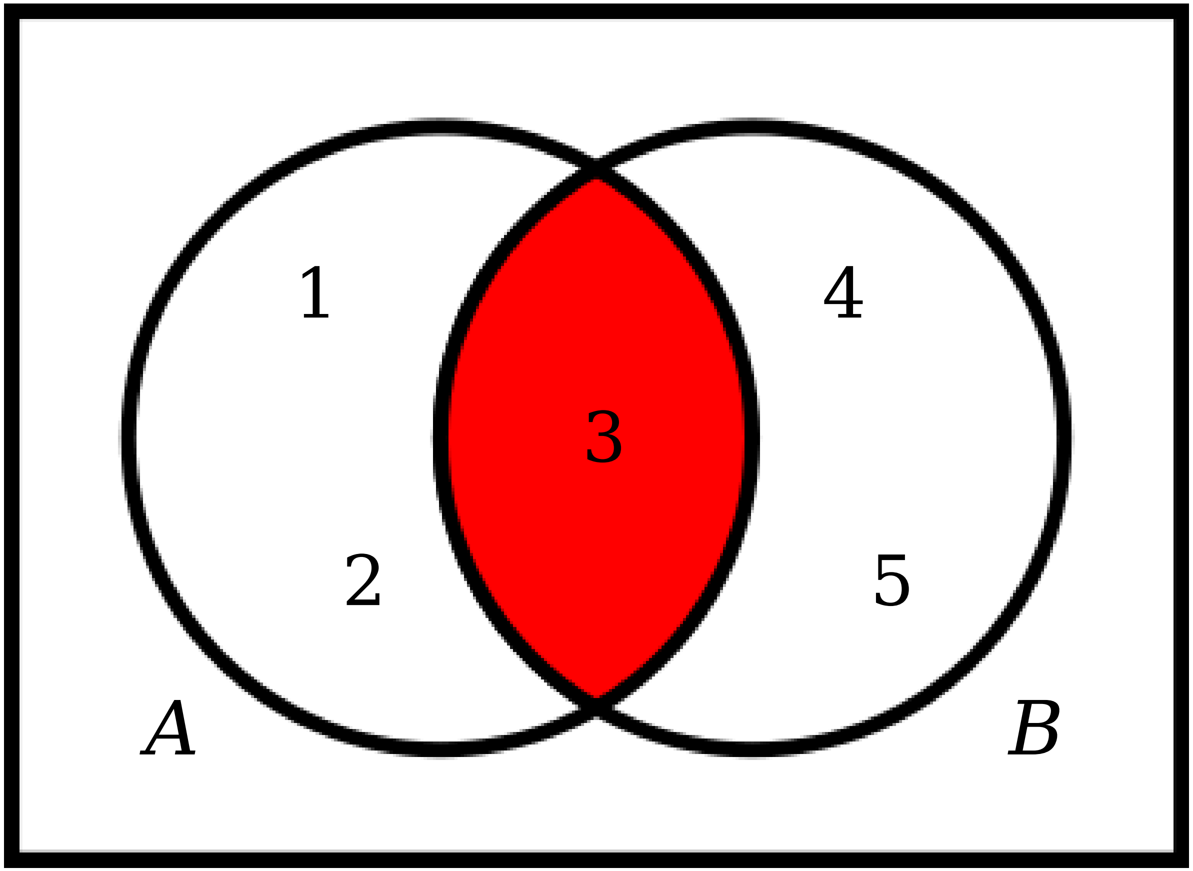
$A \cup B$

{ 1, 2, 3, 4, 5 }

$A = \{ 1, 2, 3 \}$

$B = \{ 3, 4, 5 \}$

Venn Diagrams



$A = \{ 1, 2, 3 \}$
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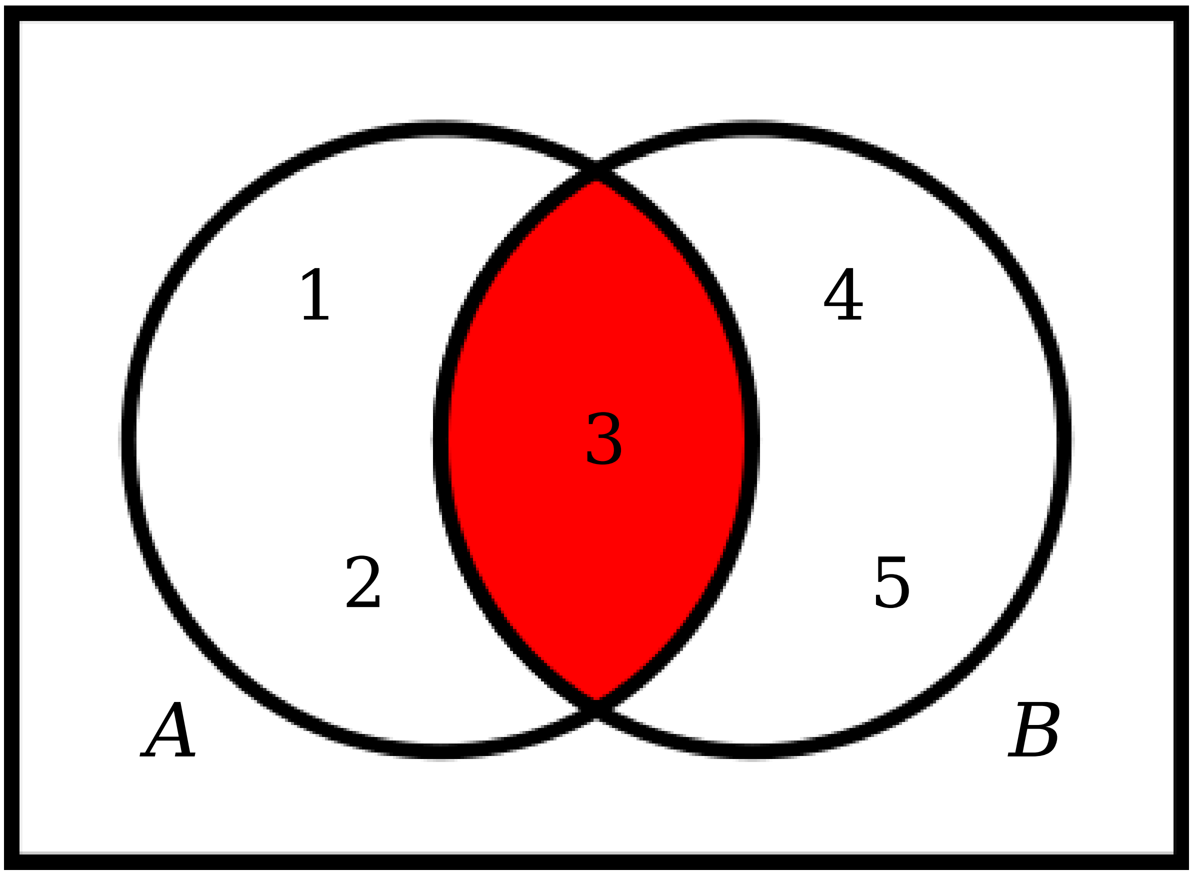
Venn Diagrams



Intersection

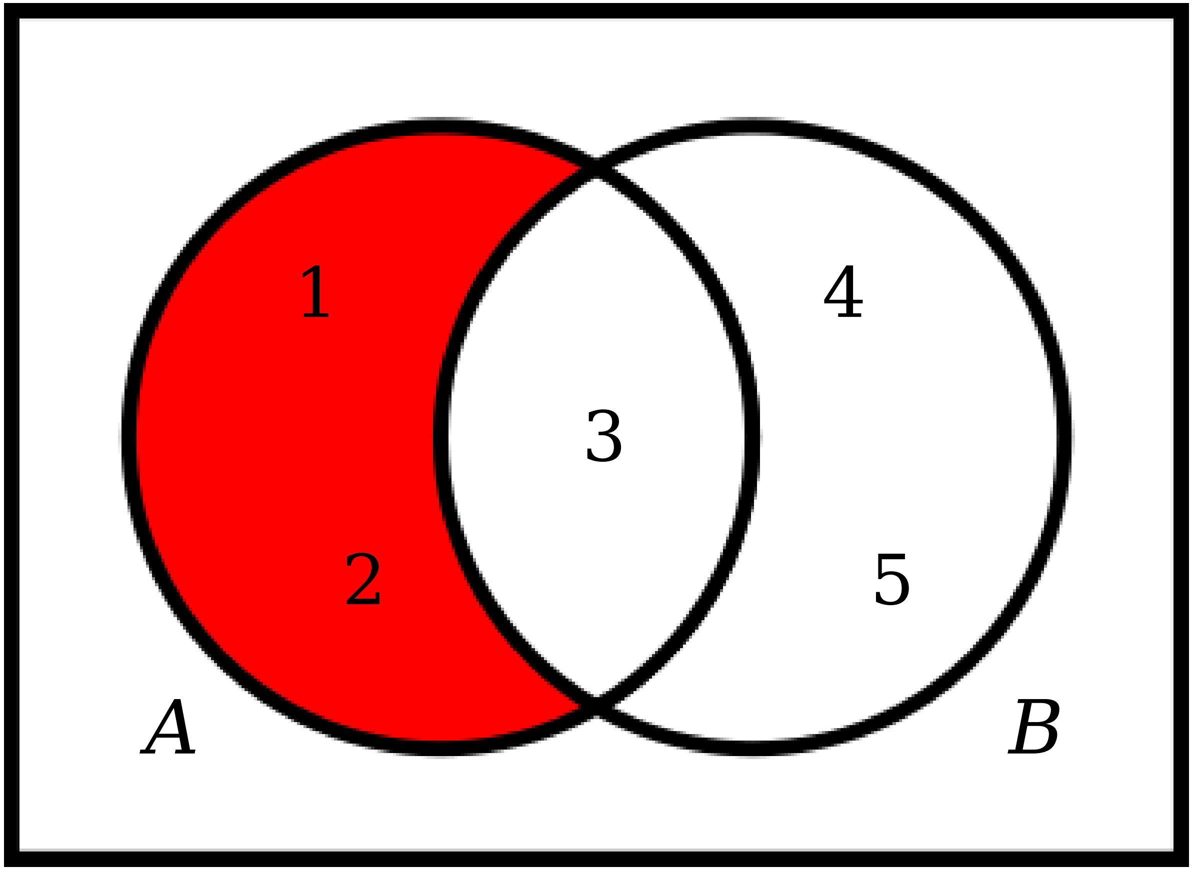
$A \cap B$

$\{ 3 \}$

$A = \{ 1, 2, 3 \}$

$B = \{ 3, 4, 5 \}$

Venn Diagrams



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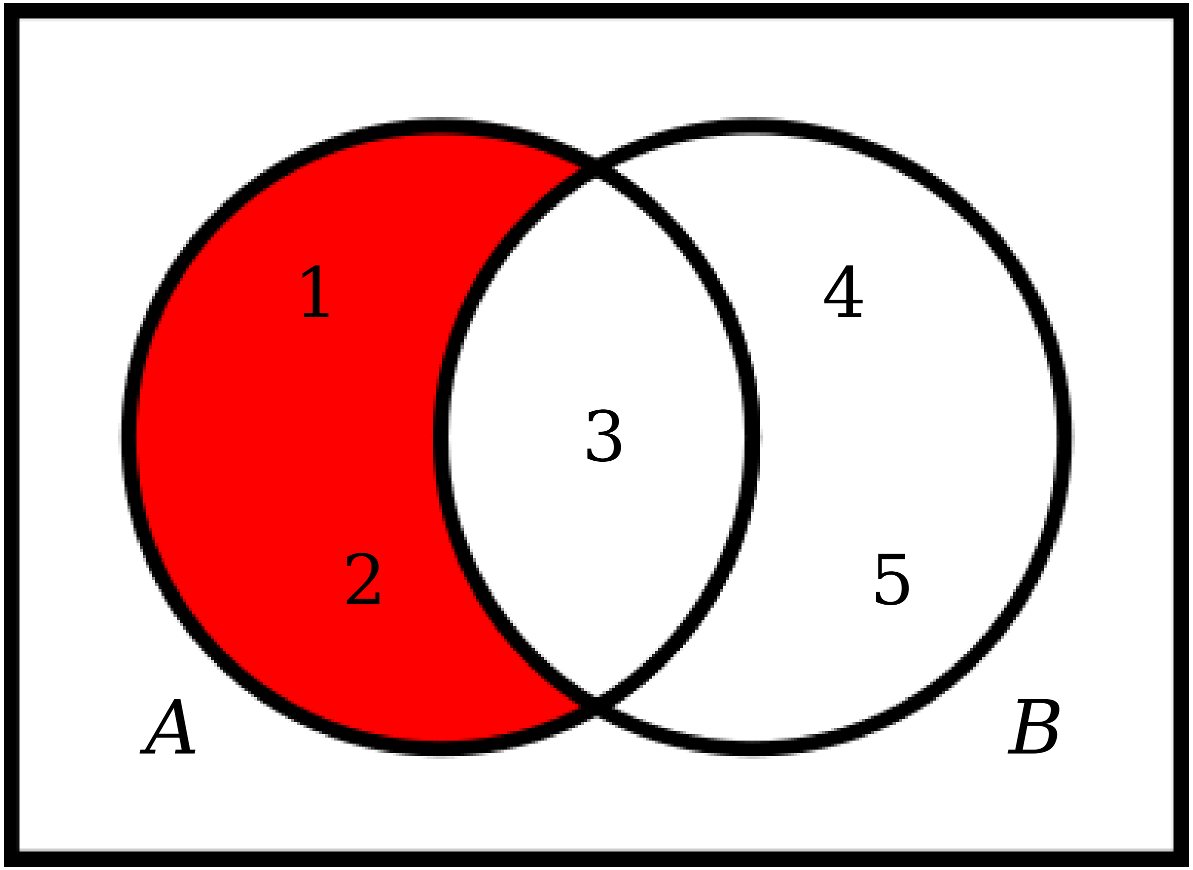
Venn Diagrams



Difference

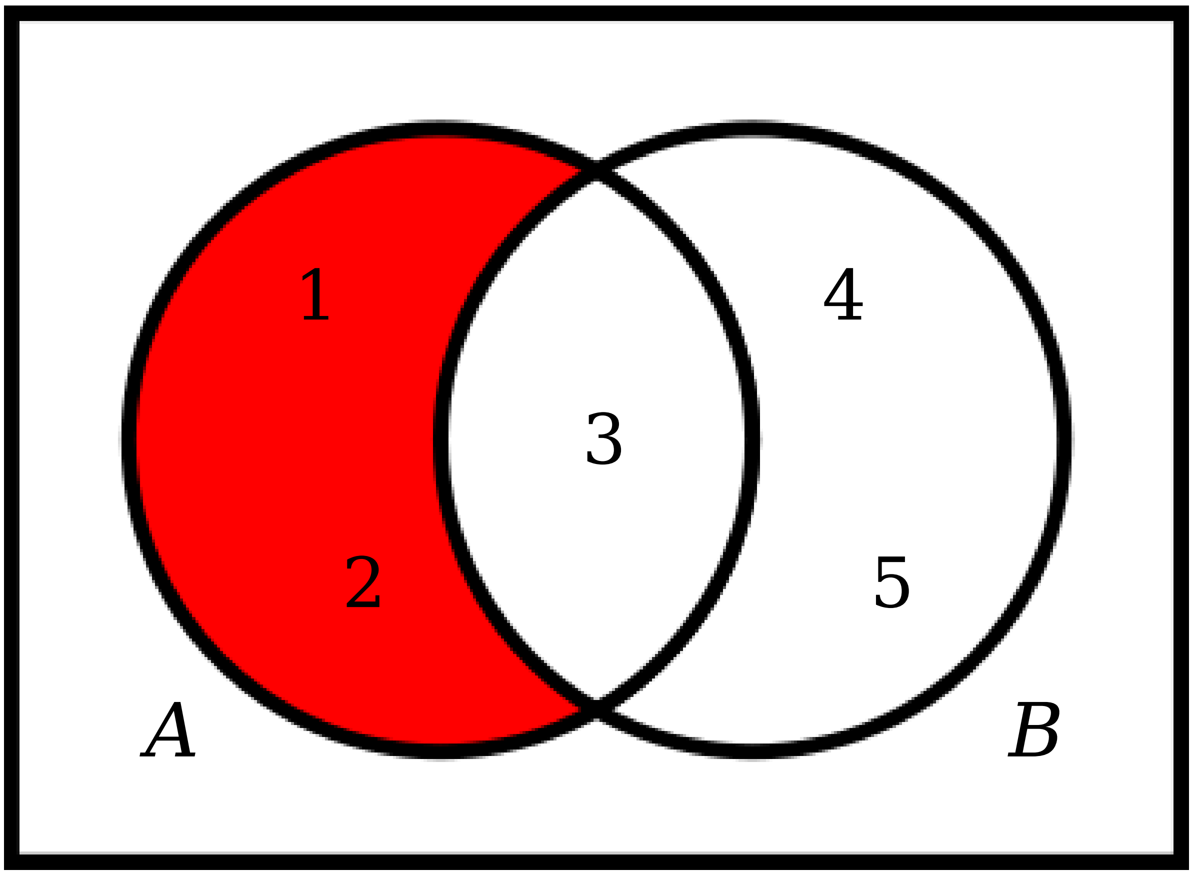
$A - B$

$\{ 1, 2 \}$

$A = \{ 1, 2, 3 \}$

$B = \{ 3, 4, 5 \}$

Venn Diagrams



Difference

$A \setminus B$

{ 1, 2 }

$A = \{ 1, 2, 3 \}$
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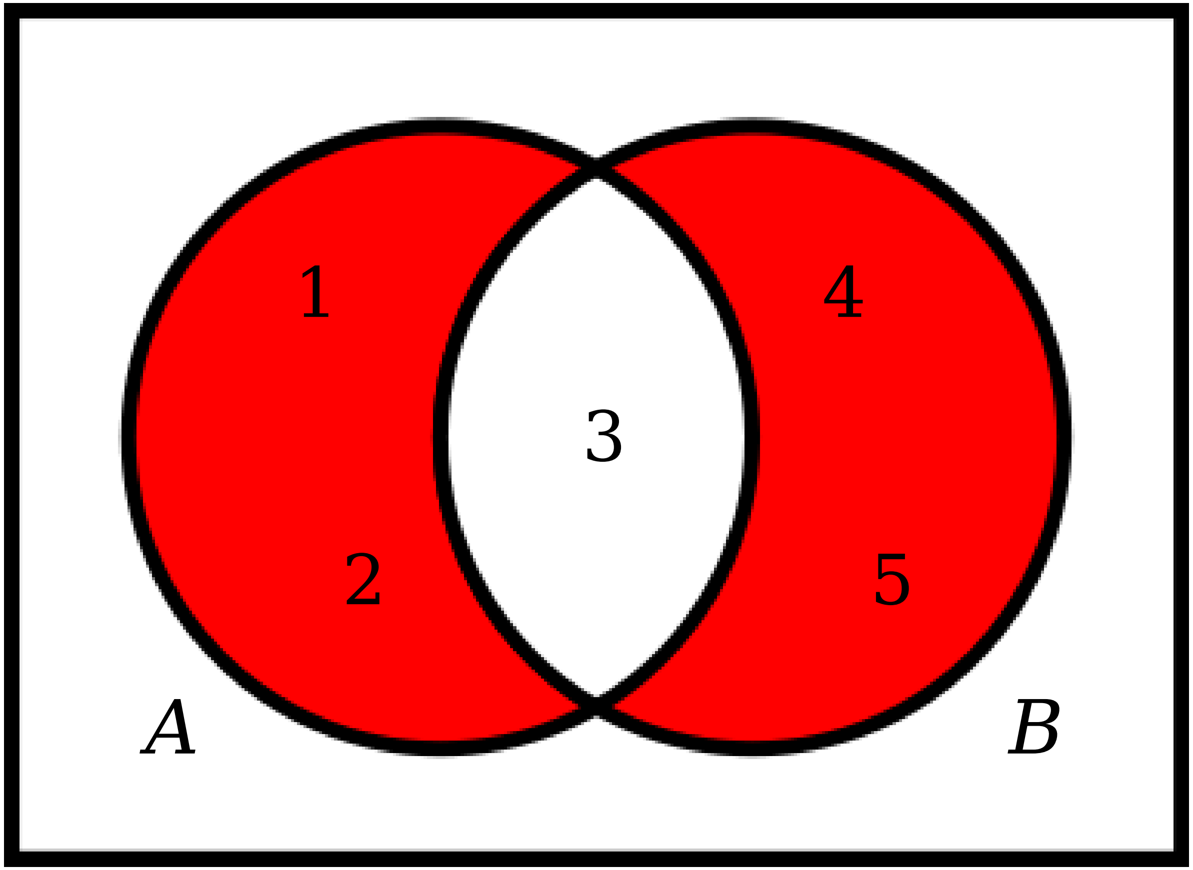
Venn Diagrams



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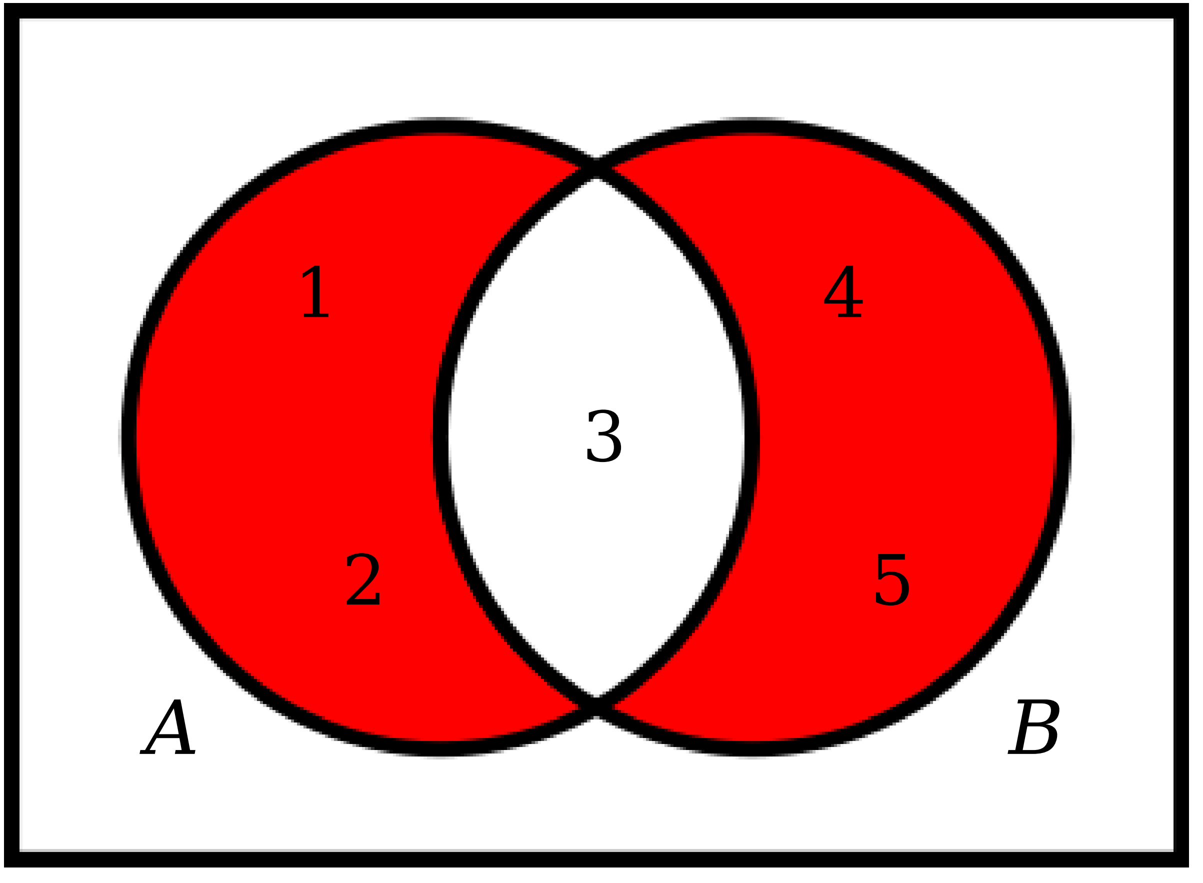

Symmetric Difference
$A \Delta B$
$\{ 1, 2, 4, 5 \}$

$A = \{ 1, 2, 3 \}$
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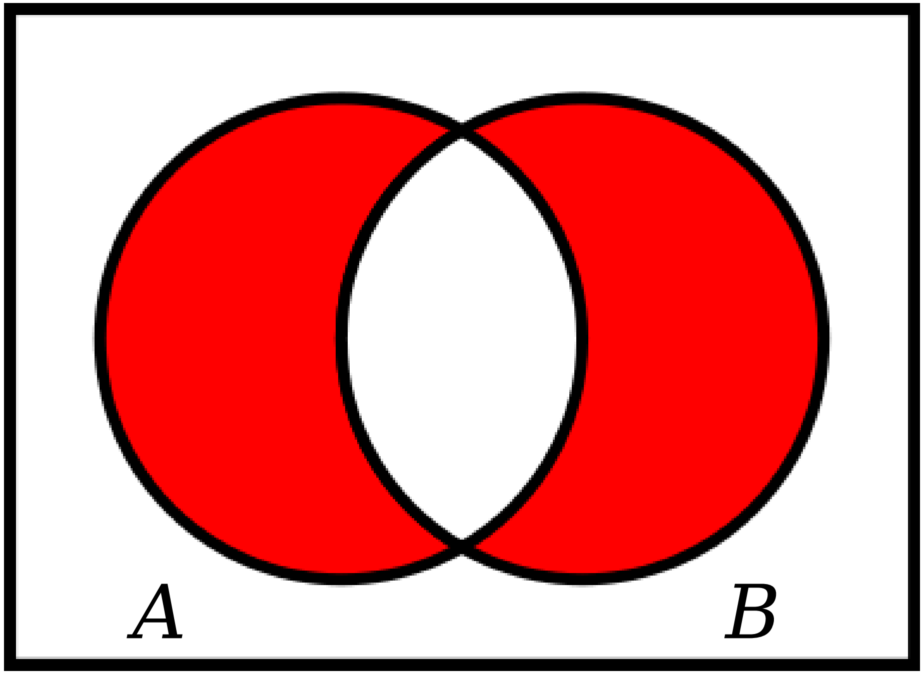
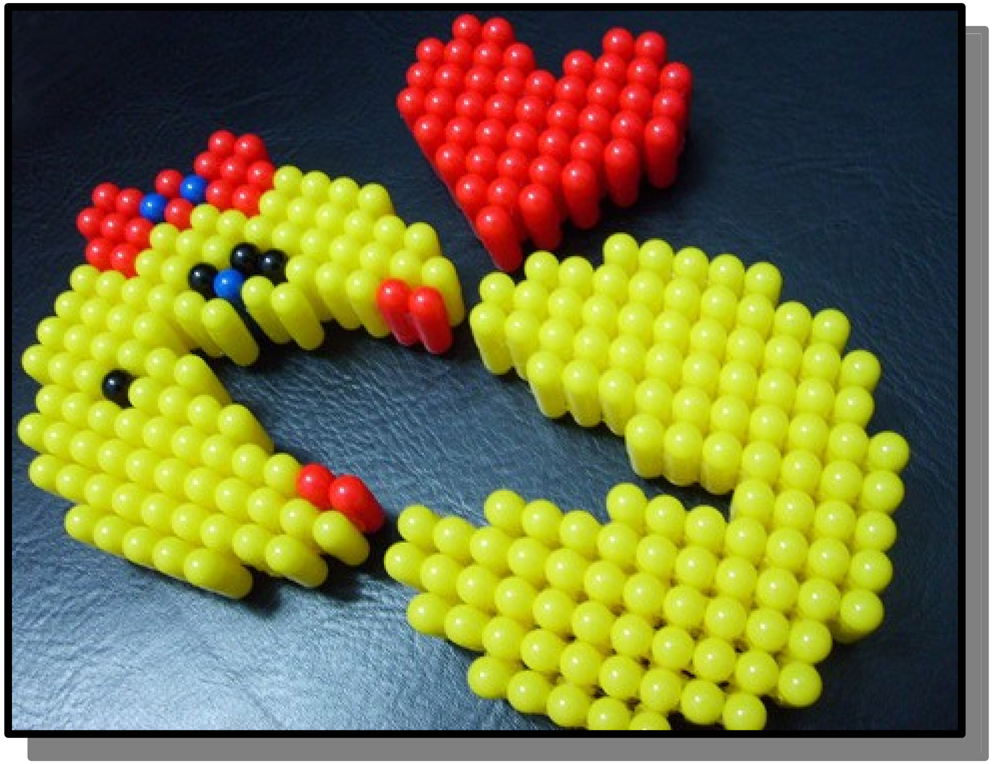
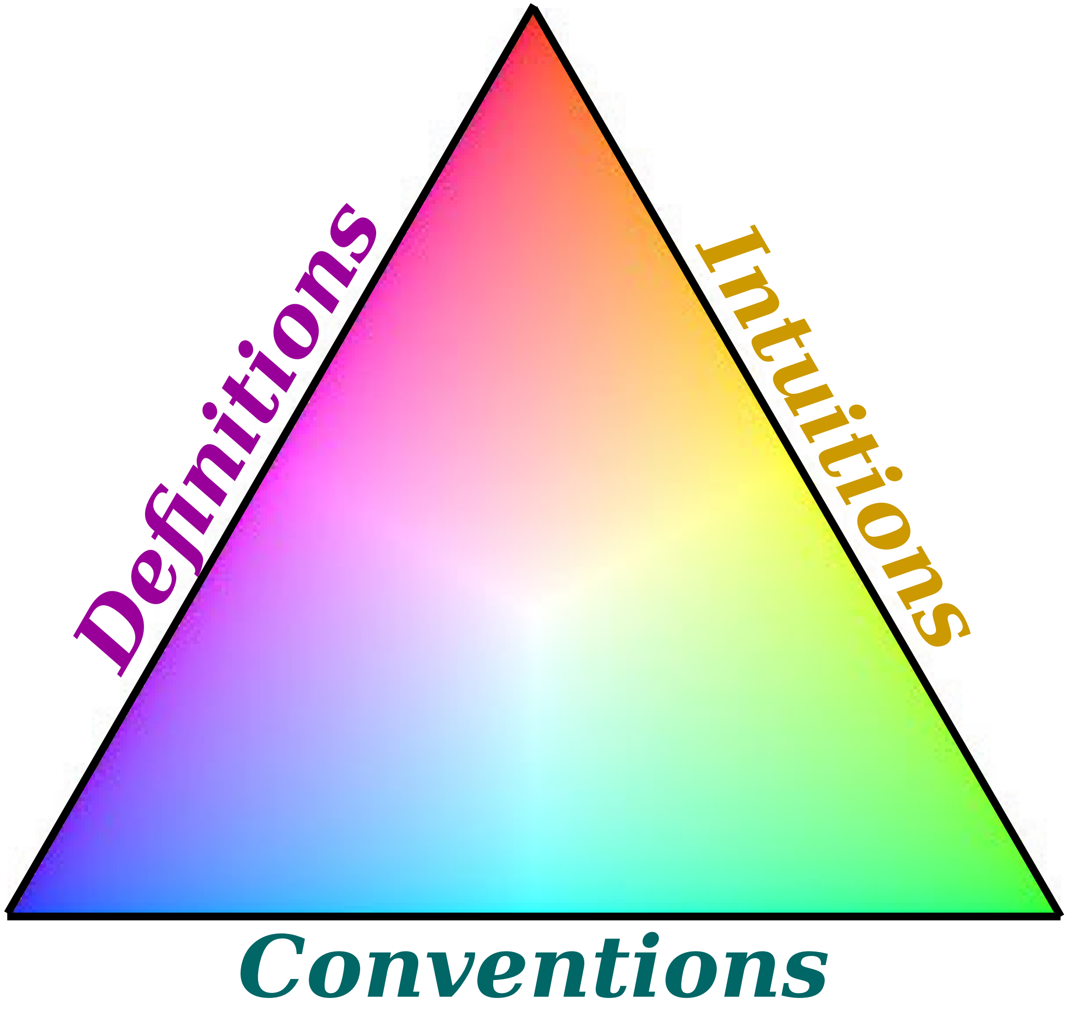
Venn Diagrams



$A \Delta B$



To Recap



Writing a good proof requires a blend of definitions, intuitions, and conventions.

An integer n is ***even*** if there is an integer k where $n = 2k$.

An integer n is ***odd*** if there is an integer k where $n = 2k+1$.

Definitions tell us what we need to do in a proof.
Many proofs directly reference these definitions.

Let's Draw Some Pictures!

Let's Do Some Math!

Let's Try Some Examples!

Building intuition for results requires creativity, trial, and error.

- Prove universal statements by making arbitrary choices.
- Prove existential statements by making concrete choices.
- Prove "If P, then Q" by assuming P and proving Q.
- Write in complete sentences.
- Number sub-formulas when referring to them.
- Summarize what was shown in proofs by cases.
- Articulate your start and end points.

Mathematical proofs have established conventions that increase rigor and readability.

Your Action Items

- ***Read "Guide to ∈ and ⊆," and "Guide to Partners."***
  - These will be very useful for PS1.
- ***Finish and submit Problem Set 0.***
  - Don't put this off until the last minute!

Next Time

- ***Indirect Proofs***
  - How do you prove something without actually proving it?
- ***Mathematical Implications***
  - What exactly does "if *P*, then *Q*" mean?
- ***Proof by Contrapositive***
  - A helpful technique for proving implications.
- ***Proof by Contradiction***
  - Proving something is true by showing it can't be false.

Appendix: ***Proofs as Dialogs***

Proofs as a Dialog

Let n be an arbitrary odd integer.

Since n is an odd integer, there is an integer k such that $n = 2k + 1$.

Now, let $z = k - 34$.

Proofs as a Dialog

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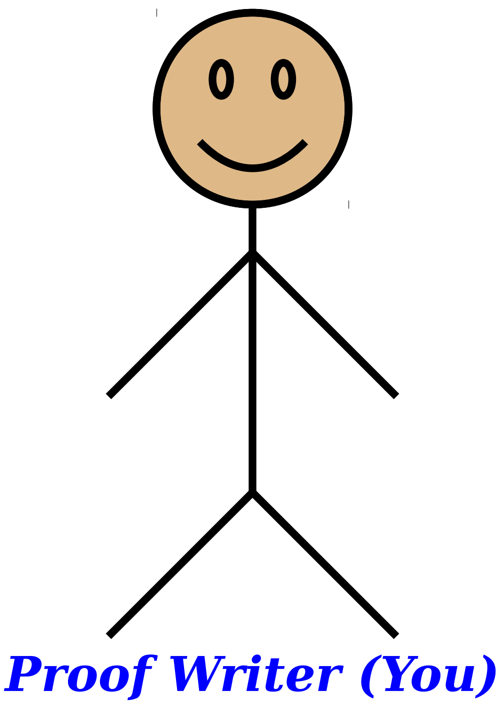

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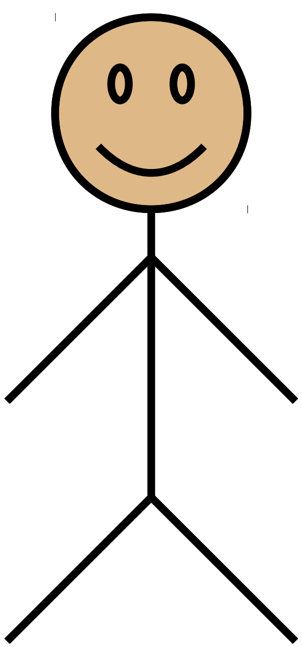
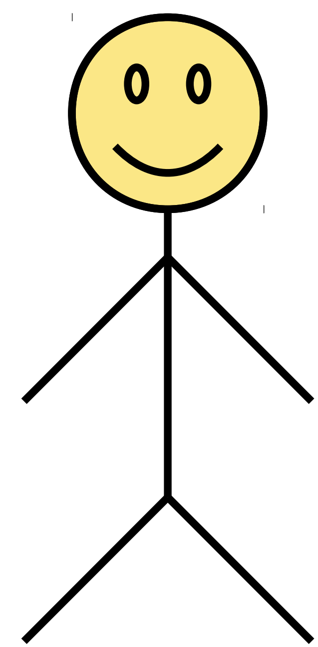


Proof Writer (You)



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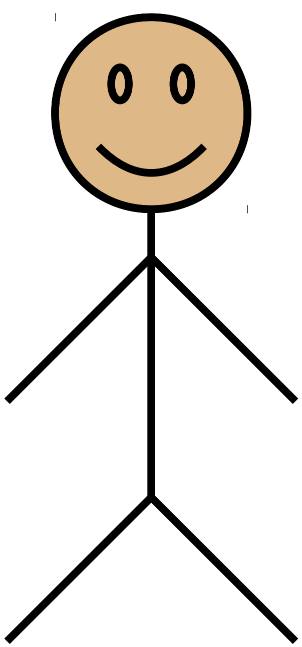

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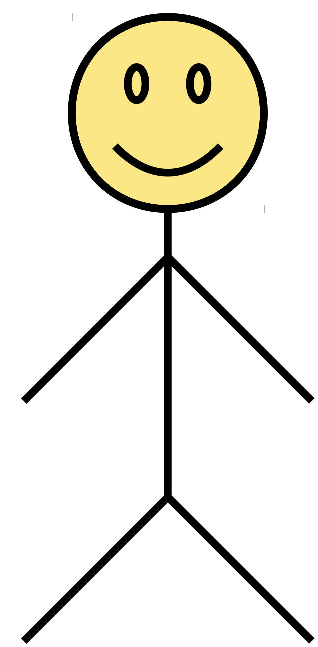

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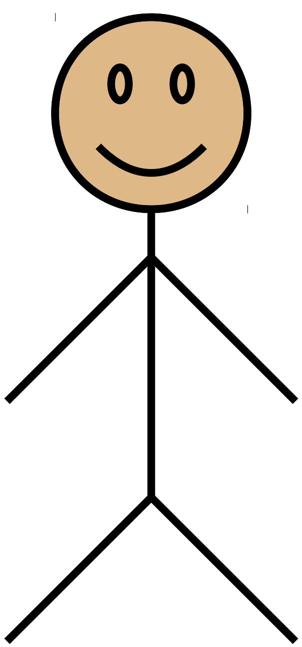
$n = 137$

Reader *Picks*



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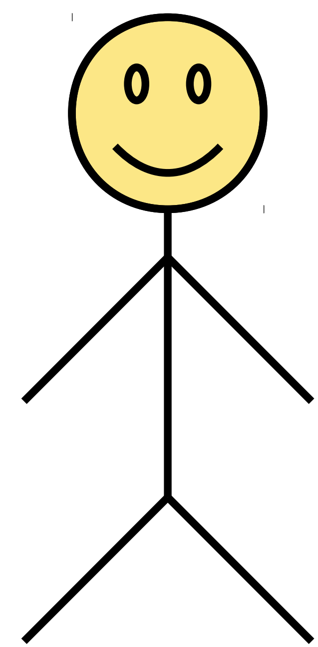

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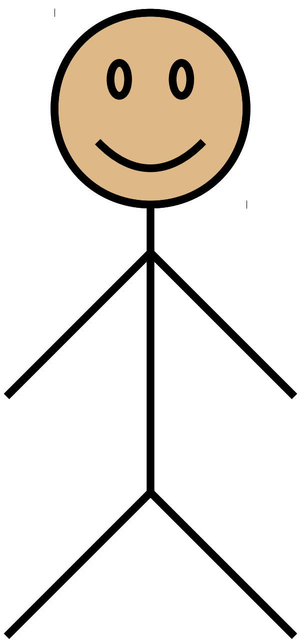

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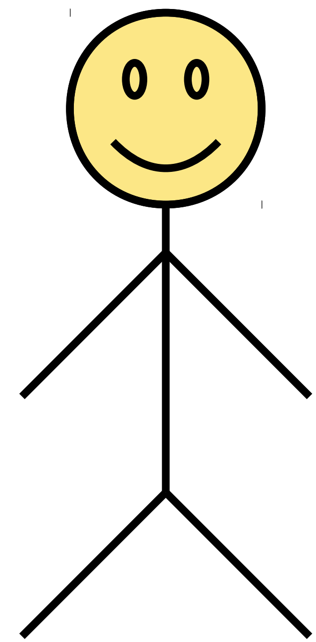

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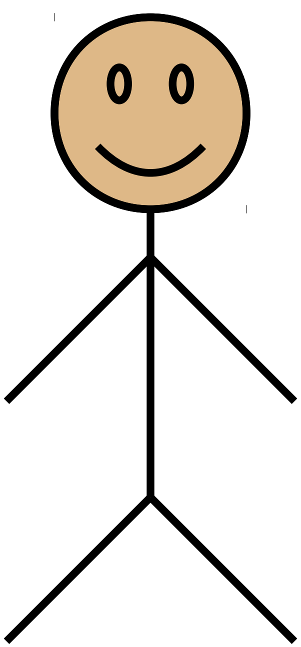
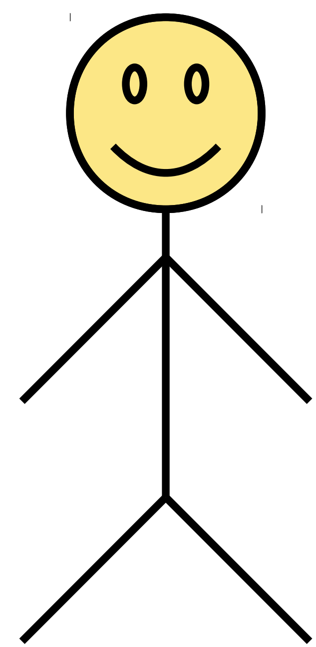


$n = 137$

Reader *Picks*

$k = 68$

Neither *Picks*



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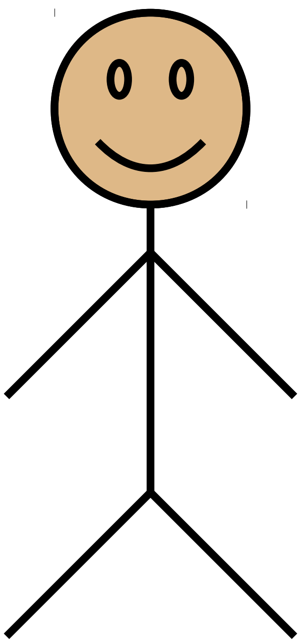


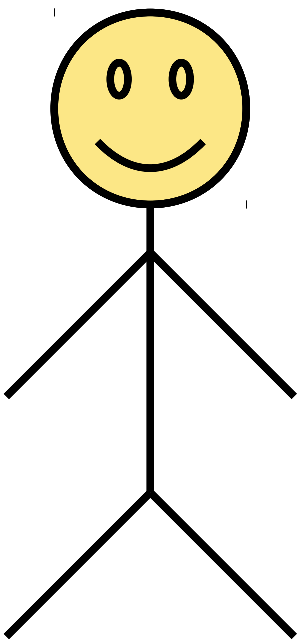
Proof Writer (You)

$k = 68$

Neither *Picks*

$n = 137$

Reader *Picks*



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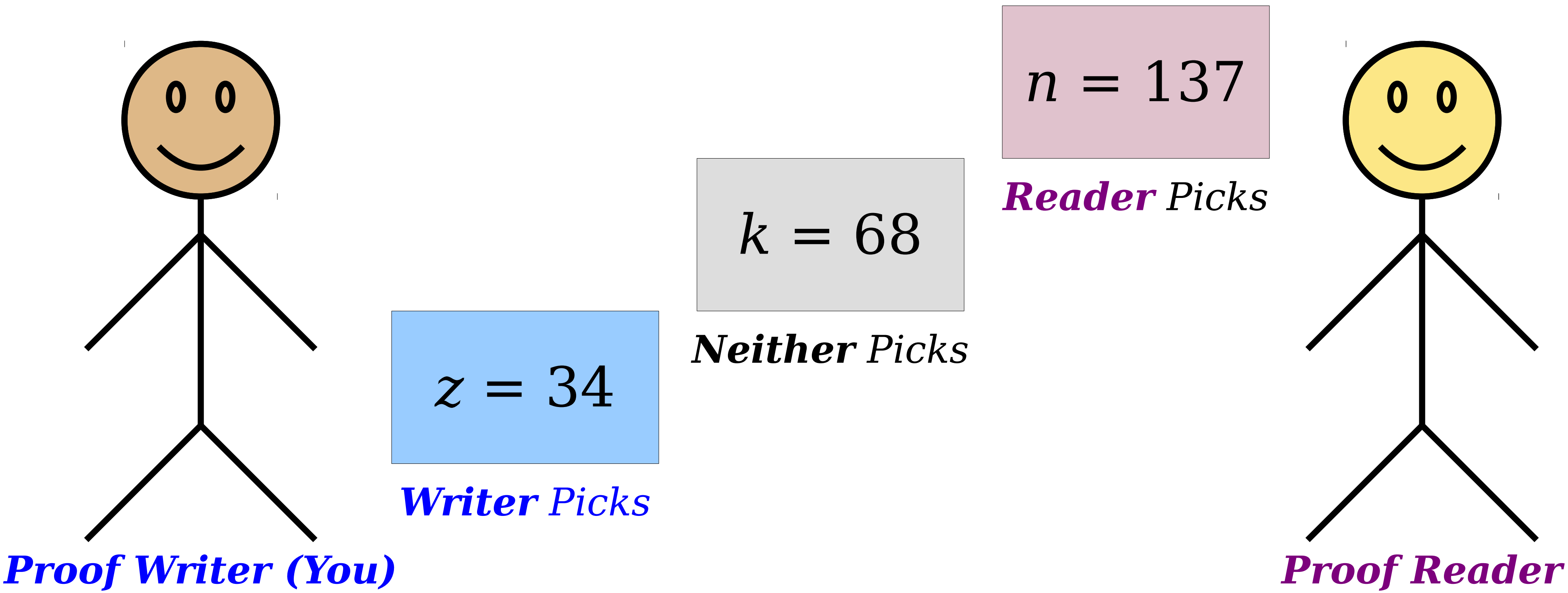

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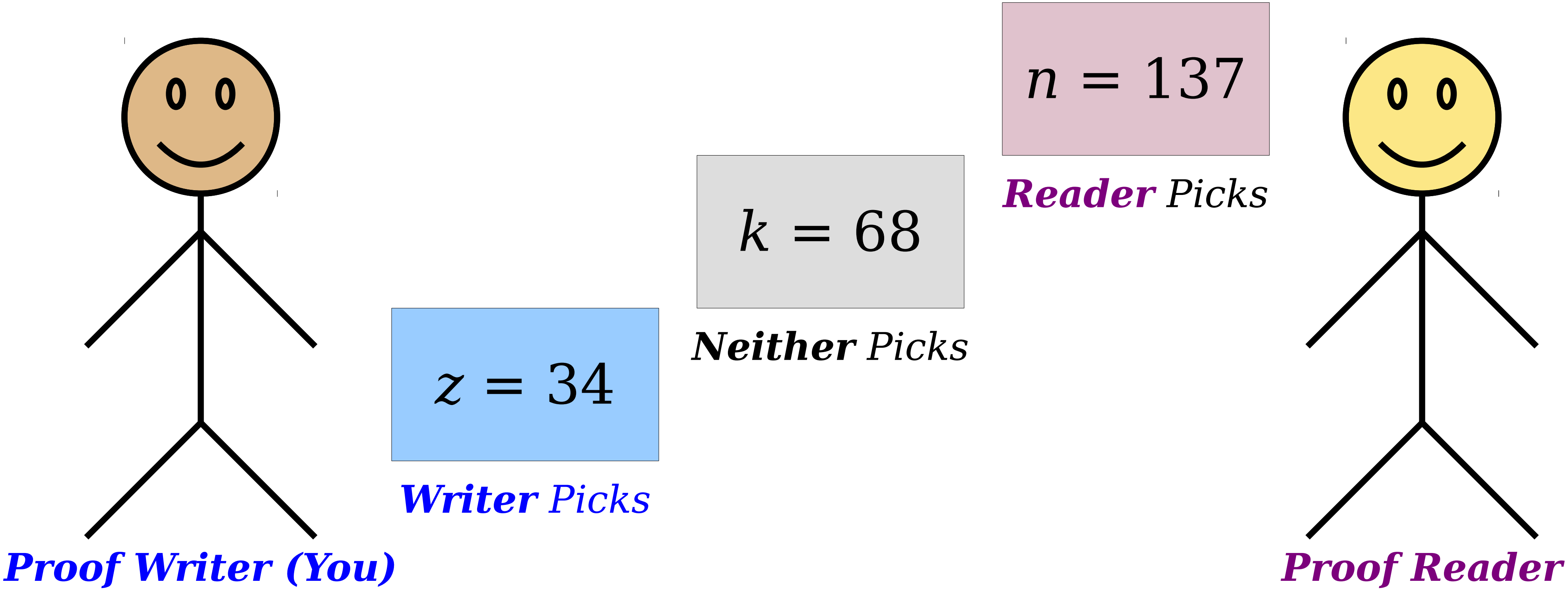

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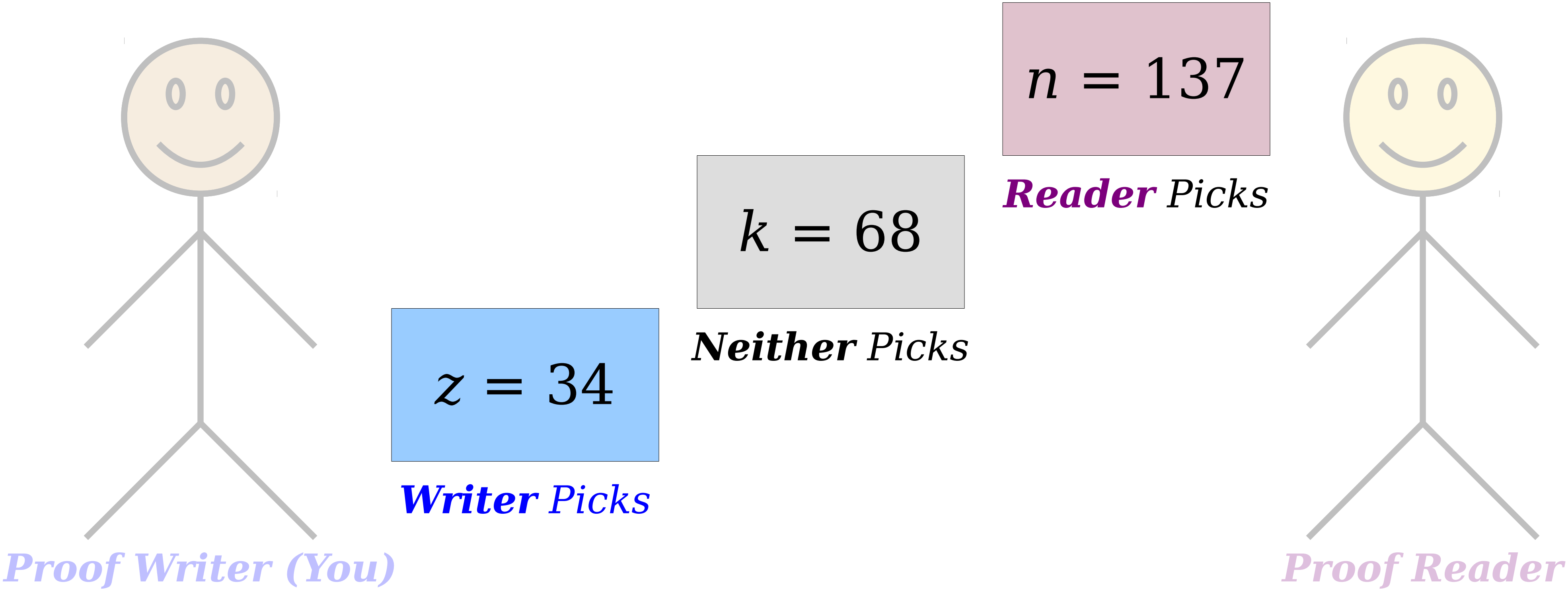


Each of these variables has a distinct, assigned value.

Each variable was either picked by the reader, picked by the writer, or has a value that can be determined from other variables.

Since … r

Now, let $z = k - 34$.

$n = 137$

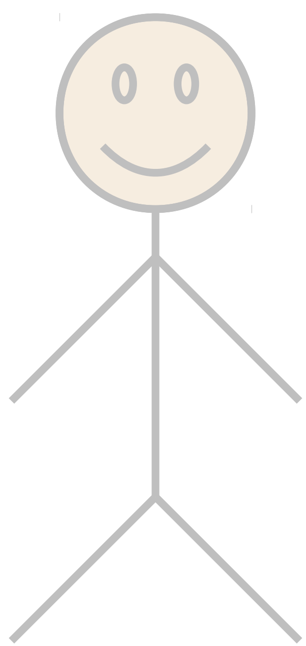
***Reader** Picks*

$k = 68$

***Neither** Picks*

$z = 34$

***Writer** Picks*



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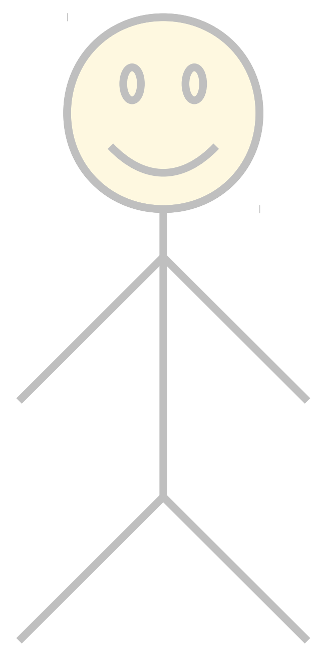


Proof Reader

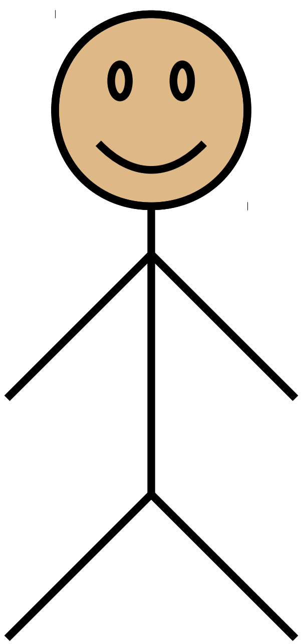
Who Owns What?

- The ***reader*** chooses and owns a value if you use wording like this:
  - Pick a natural number n.
  - Consider some $n \in \mathbb{N}$.
  - Fix a natural number n.
  - Let n be a natural number.
- The ***writer*** (you) chooses and owns a value if you use wording like this:
  - Let $r = n + 1$.
  - Pick $s = n$.
- ***Neither*** of you chooses a value if you use wording like this:
  - Since n is even, we know there is some $k \in \mathbb{Z}$ where $n = 2k$.
  - Because n is odd, there must be some integer k where $n = 2k + 1$.

Proofs as a Dialog

⚠ Let x be an arbitrary even integer. ⚠

Then for any even x, we know that $x+1$ is odd.



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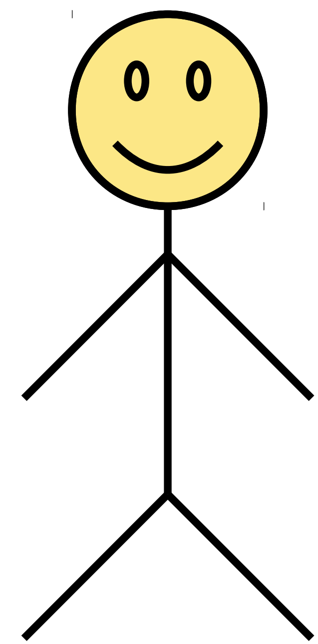

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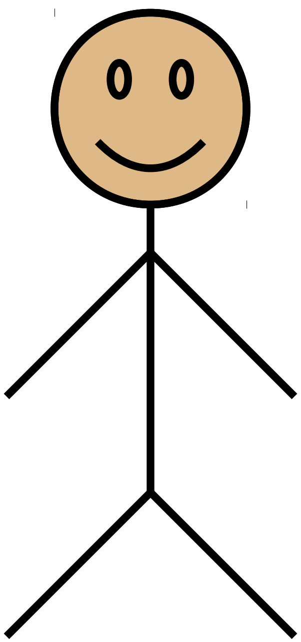

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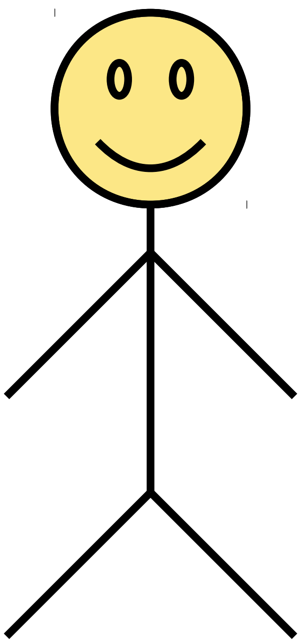

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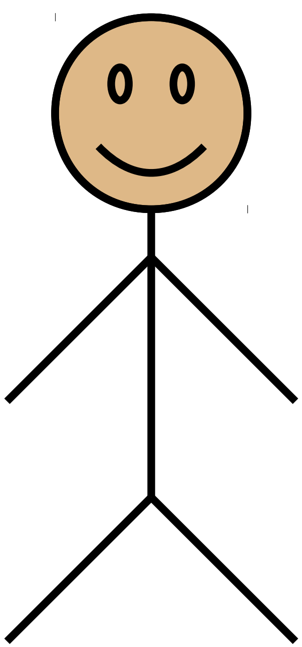
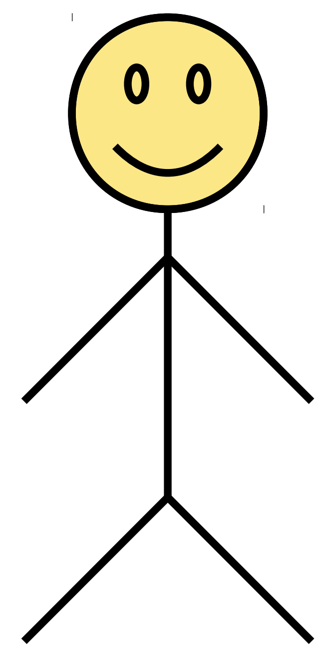


$x = 242$

Reader *Picks*



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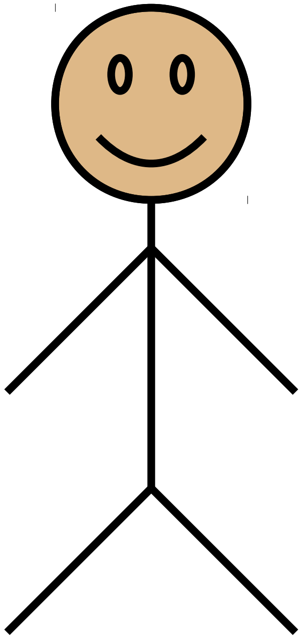

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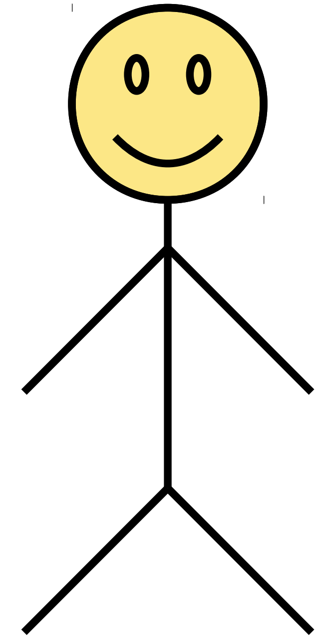

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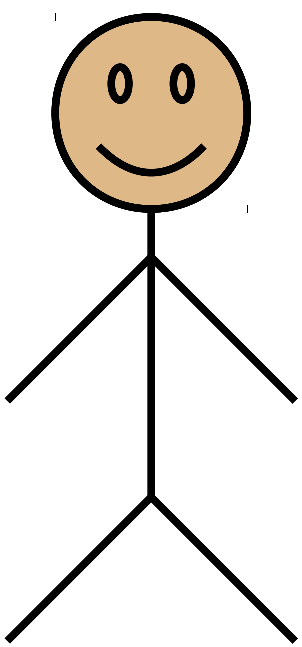

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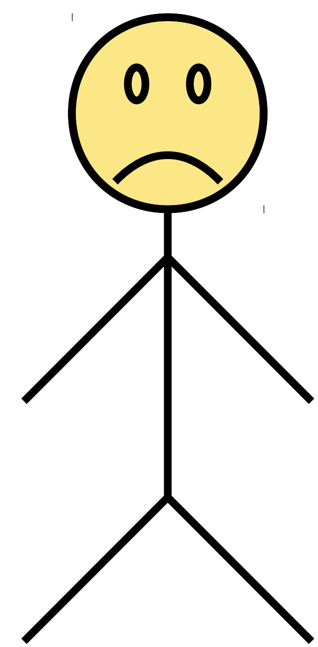

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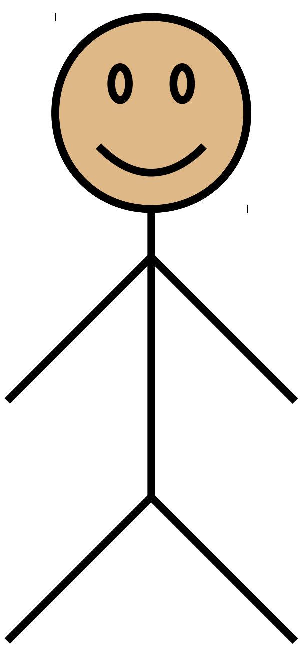

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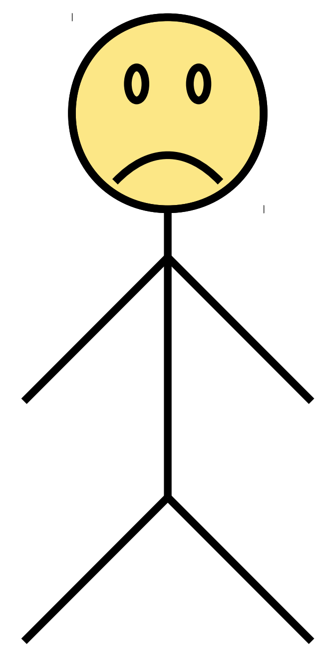

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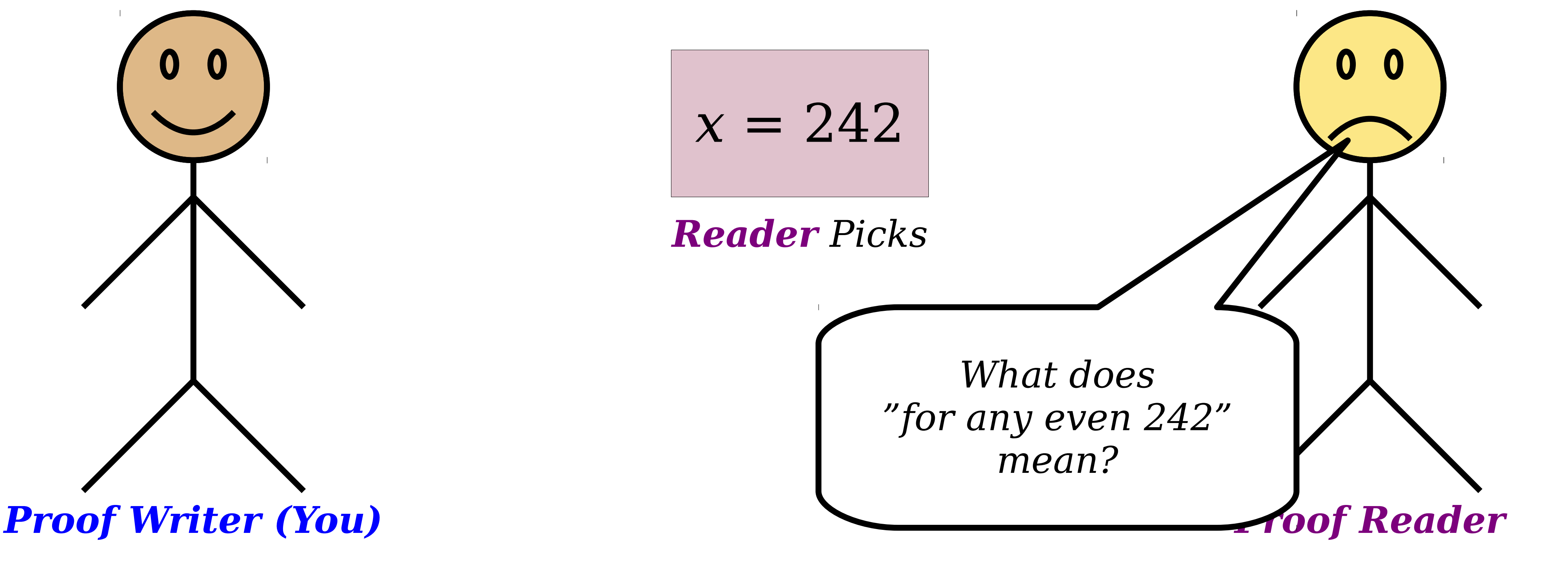

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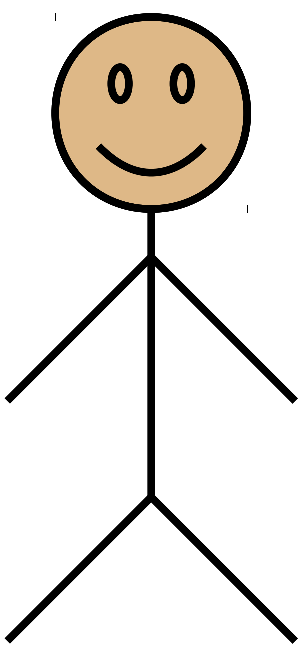

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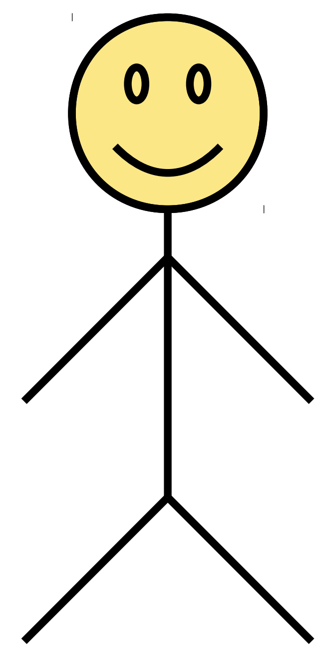

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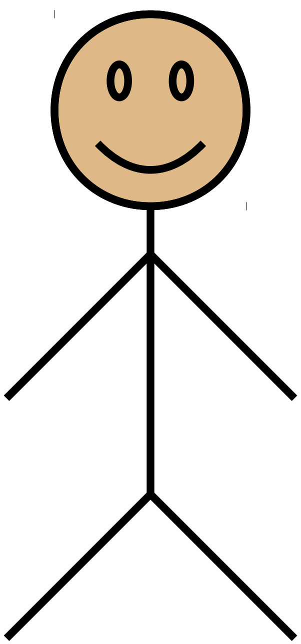

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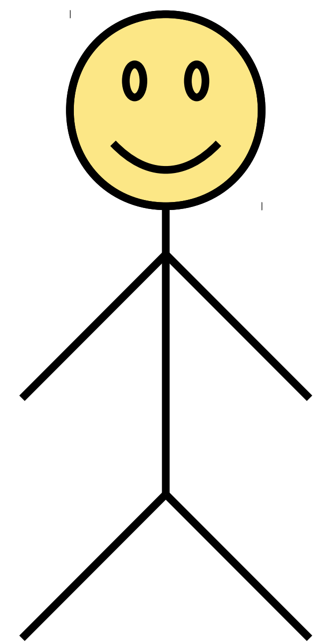

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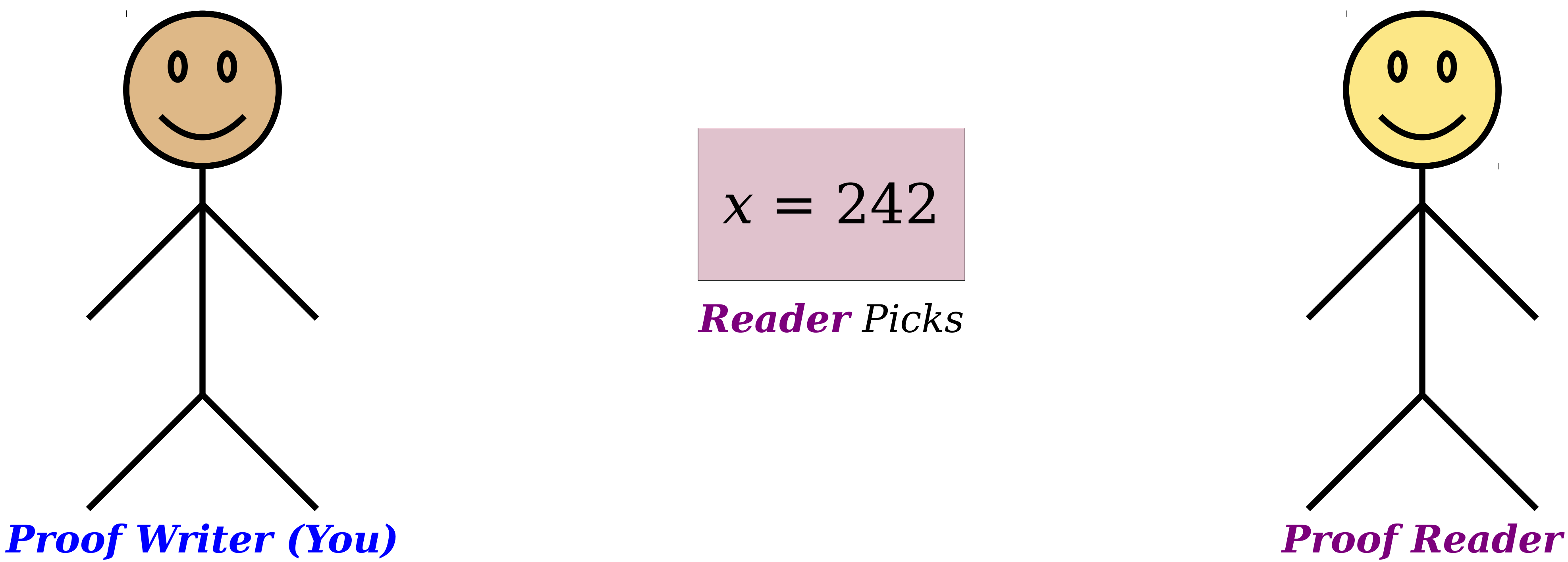

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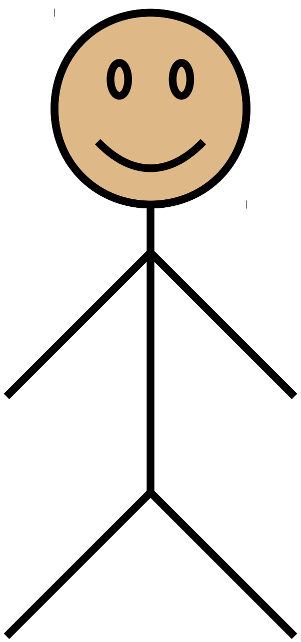
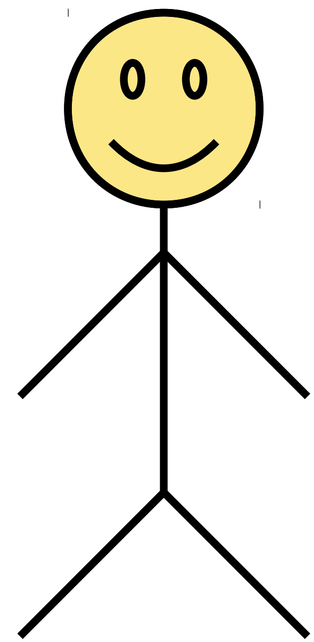


Proof Writer (You)

$x = 242$

Reader *Picks*



Proof Reader

Every variable needs a value.

Avoid talking about "all x" or "every x" when manipulating something concrete.

To prove something is true for any choice of a value for x, let the reader pick x.

Once you've said something like

Let x be an integer.

Consider an arbitrary $x \in \mathbb{Z}$.

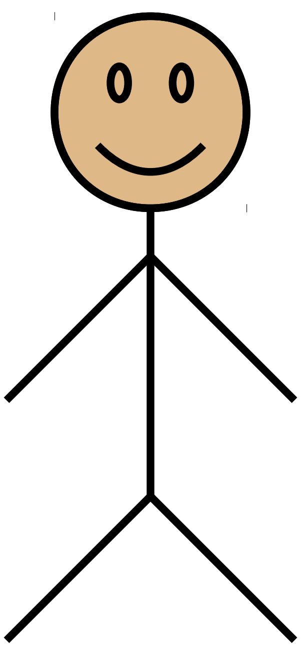
Pick any x.

Do not say things like the following:

This means that ***for any*** $x \in \mathbb{Z}$ ...

So ***for all*** $x \in \mathbb{Z}$ ...

Proofs as a Dialog



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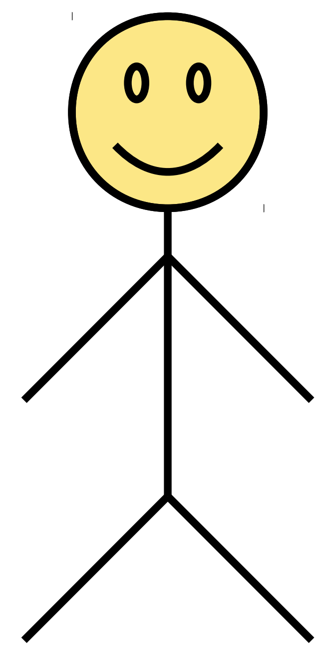


Proof Reader

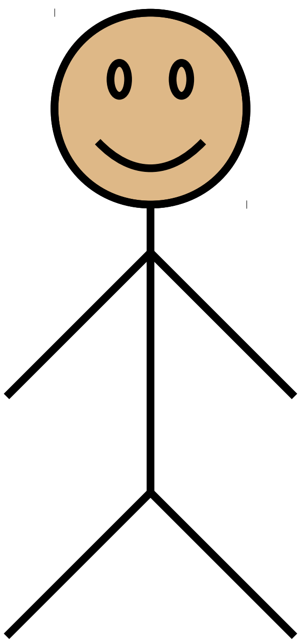
Proofs as a Dialog

⚠

Pick two integers m and n where $m+n$ is odd.

Let $n = 1$, which means that $m+1$ is odd.

⚠



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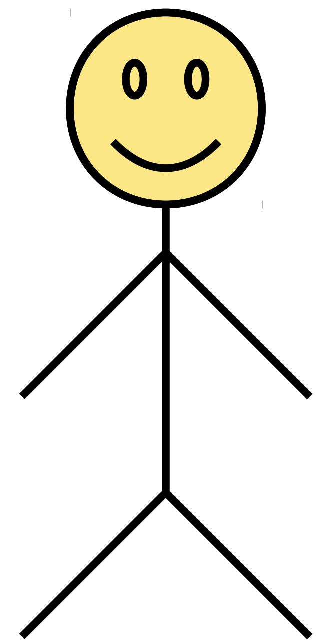

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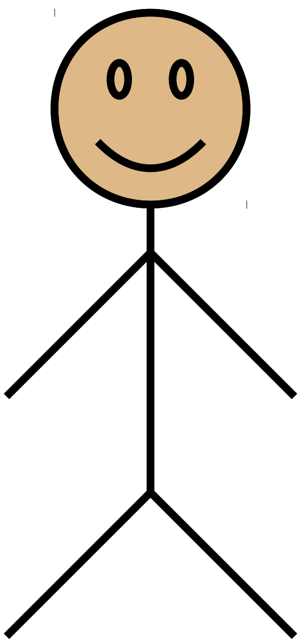

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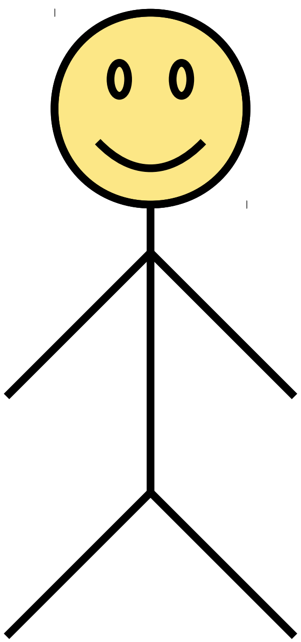

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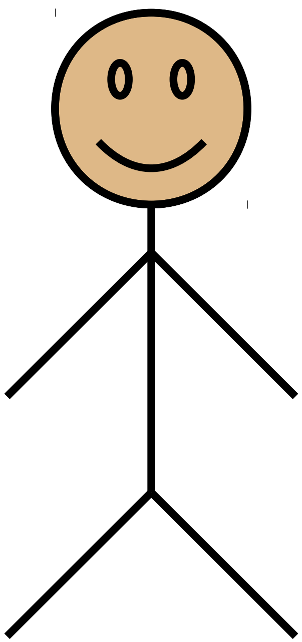


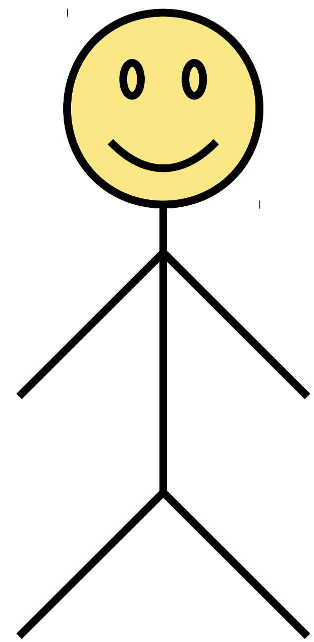
Proof Writer (You)

$m = 103$

Reader *Picks*

$n = 166$

Reader *Picks*



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Reader *Picks*

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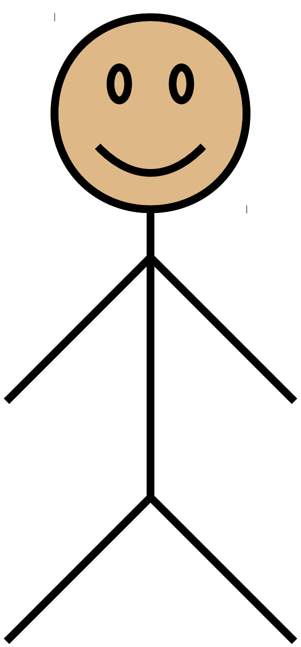
Reader *Picks*



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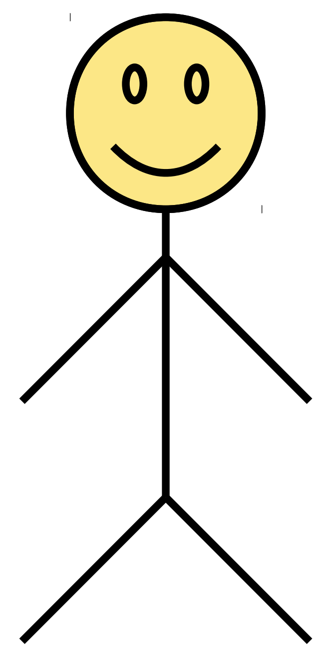

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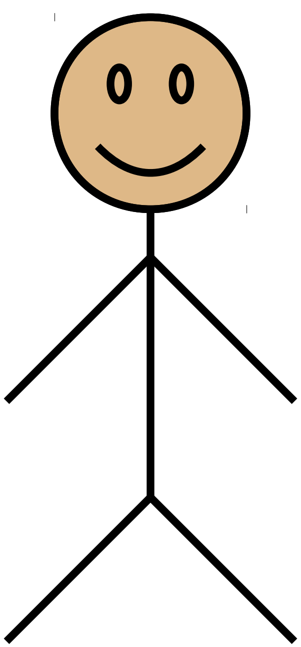

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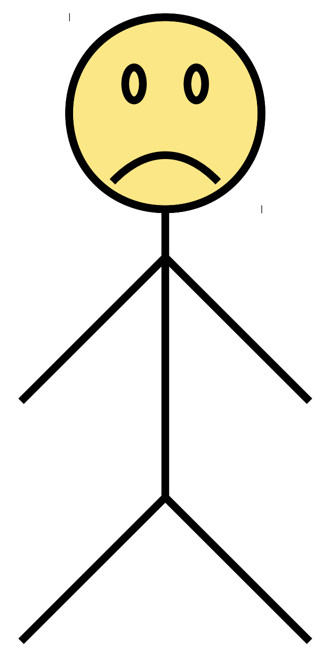

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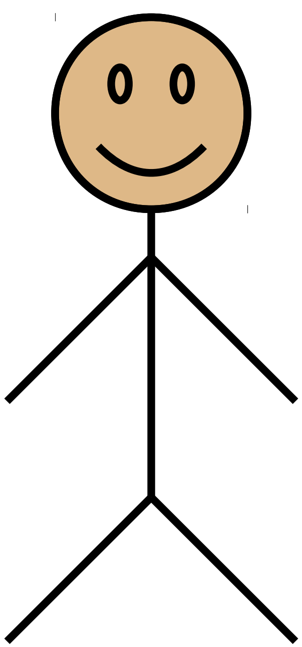

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Reader *Picks*

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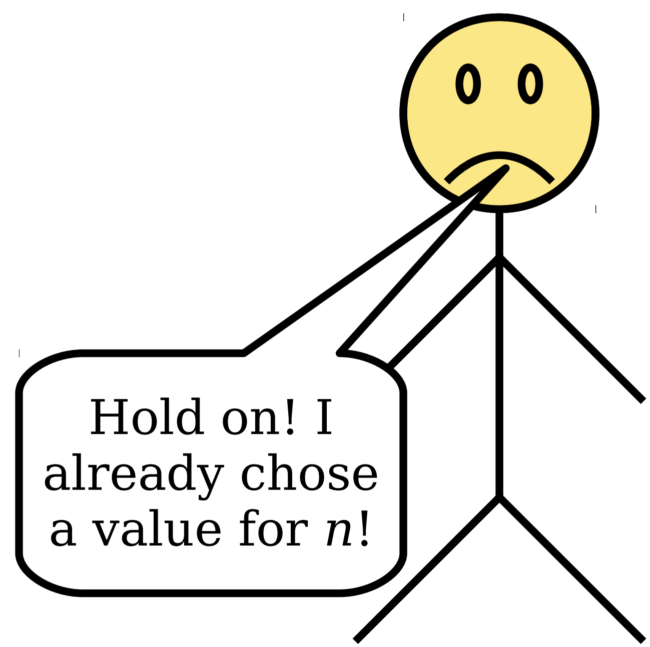
Reader *Picks*

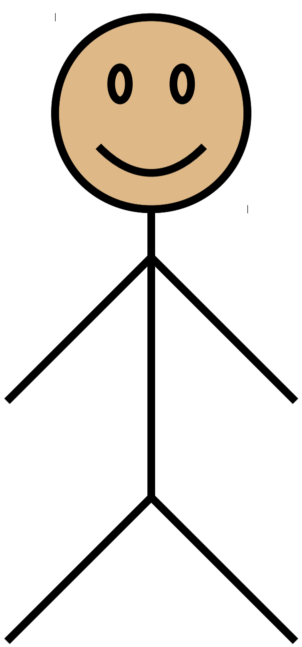


Proof Reader

Proofs as a Dialog

⚠ Let $n = 1$.

Pick any integer m where $m+1$ is odd. ⚠



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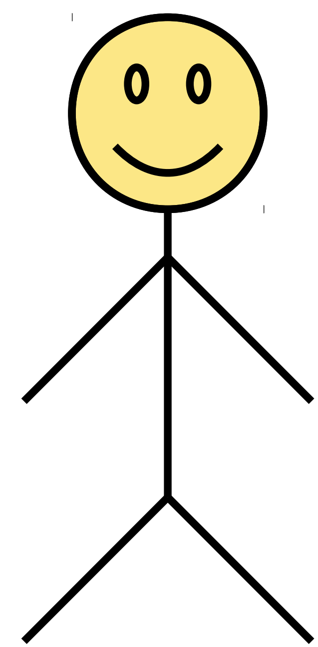
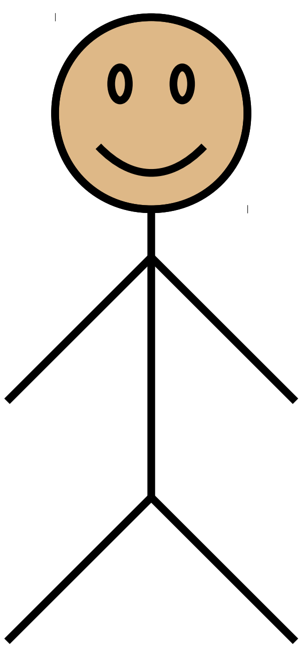


Proof Reader

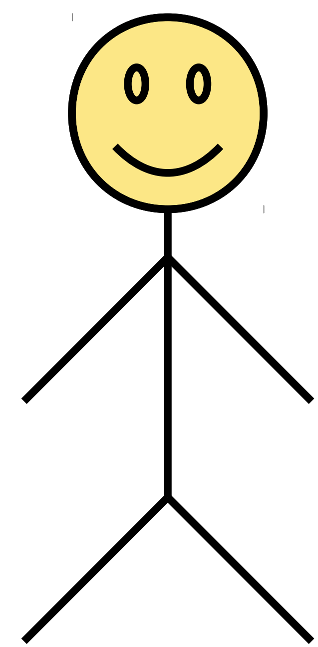
Proofs as a Dialog

⚠ Let $n = 1$. ⚠

Pick any integer m where $m+1$ is odd.



Proof Writer (You)



Proof Reader

Proofs as a Dialog

⚠

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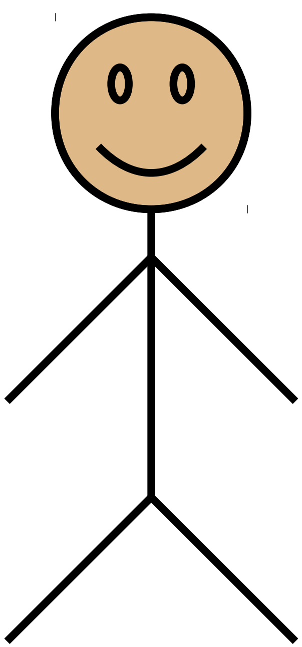
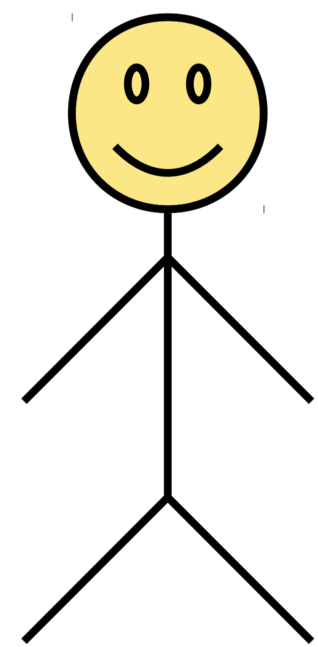
⚠



$n = 1$

Writer *Picks*



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Proof Reader

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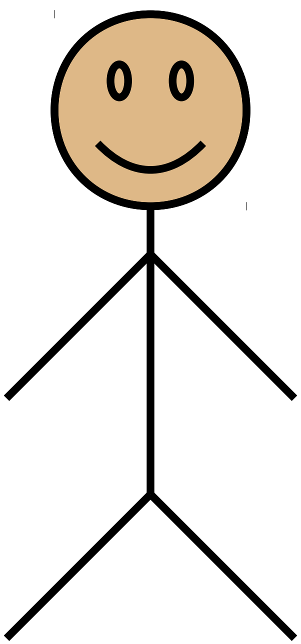
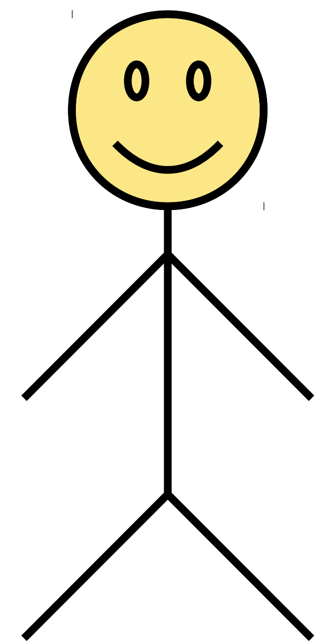


Proof Writer (You)

$n = 1$

Writer *Picks*



Proof Reader

Proofs as a Dialog

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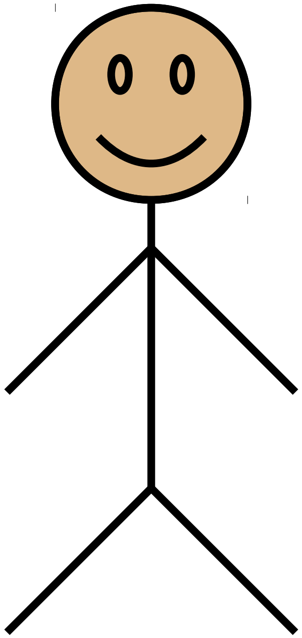
⚠



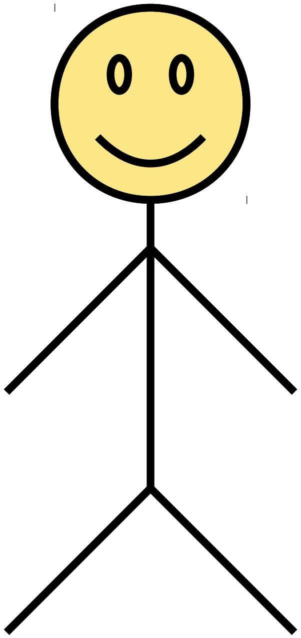
Proof Writer (You)

$m = 166$

Reader *Picks*

$n = 1$

Writer *Picks*



Proof Reader

Proofs as a Dialog

Let $n = 1$.

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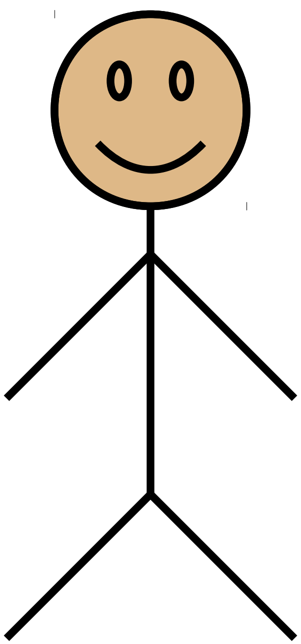


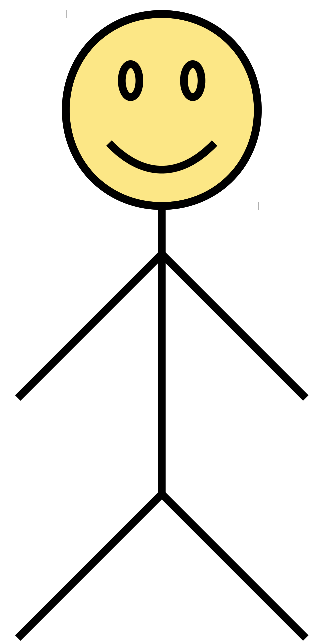
Proof Writer (You)

$m = 166$

Reader *Picks*

$n = 1$

Writer *Picks*



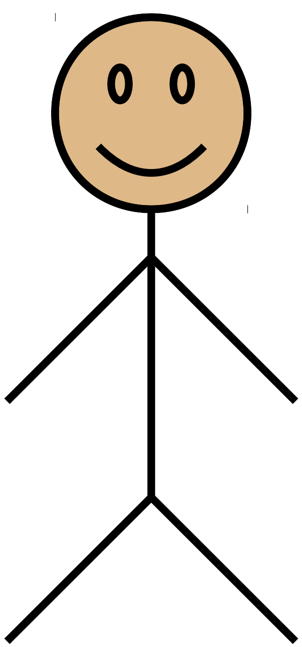
Proof Reader

Proofs as a Dialog

Let $n = 1$.

Pick any integer m where $m+1$ is odd.

Do we even need n here?

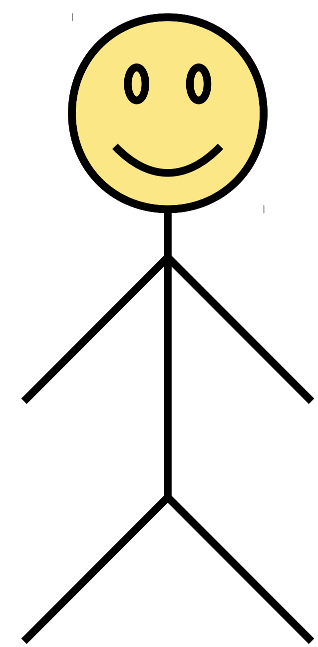


$m = 166$

Reader *Picks*

$n = 1$

Writer *Picks*



Proof Writer (You)

Proof Reader

Proofs as a Dialog

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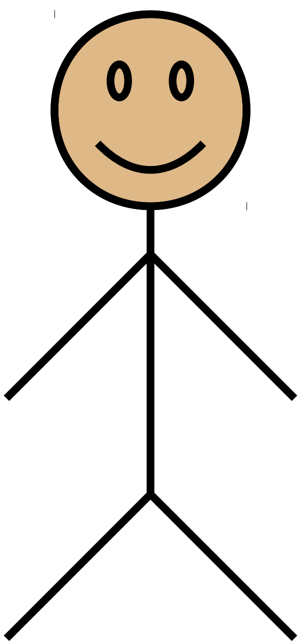
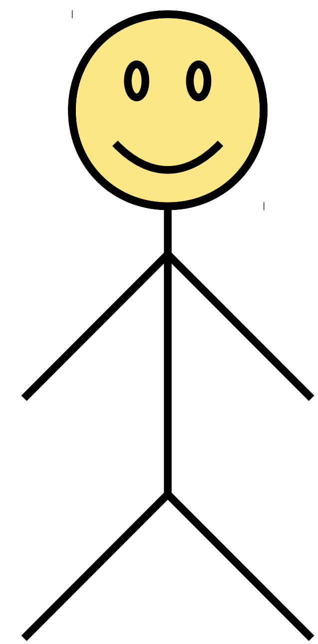


Proof Writer (You)



Proof Reader

Proofs as a Dialog

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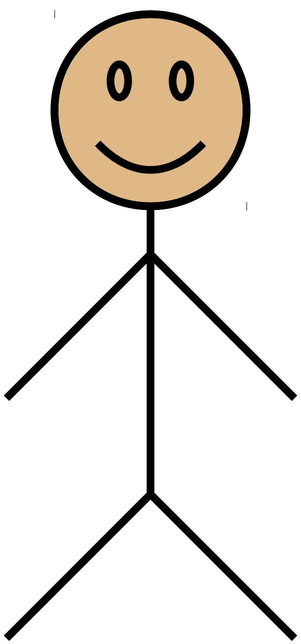
Reader *Picks*



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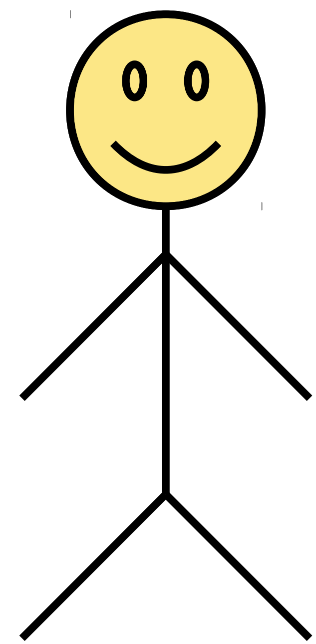

Proof Reader

Be mindful of who owns what variable.

Don’t change something you don’t own.

You don’t always need to name things, especially if they already have a name.